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THE ANALYSIS OF NONUNIFORM DISTRIBUTED
R-C STRUCTURE

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "The Analysis of Nonuniform Distributed R-C Structure" submitted by Ahmed I. A. Salama in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

The nonuniform distributed R-C structure has received renewed attention recently. Several methods have been used so far [1,2,3,4] to solve the partial differential equation with variable coefficients, which describes the structure under sinusoidal steady state conditions. The different methods have one feature in common, namely, all of them make use of some kind of transformation of the variables in the differential equation.

In this thesis a new transformation (namely, transformation of the independent variable x) has been used. The main advantage of this transformation lies in the fact that the solution of the transformed equation can be obtained very easily by comparing the transformed equation with equations whose solutions are known. Another advantage which results is that several distributed structures with the same electrical behaviour can be generated, some of which may be more suitable for fabrication. Such distributed structures are said to be equivalent.

The frequency responses of specific R-C structures with various distributions $Z(x)$, $Y(x)$ have been obtained.

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LIST OF SYMBOLS

$r(x) = Z(x)$	Spatial variation of the resistance/unit length
$c(x) = Y(x)$	Spatial variation of the capacitance/unit length
t	Time
x	Space variable (distance)
$v(x,t)$	The voltage at distance x , and time t
$i(x,t)$	The current at distance x , and time t
s	The Laplace variable
k	Frequency factor = $j\omega$
Z_ν	A linear combination of Bessel functions of order ν
W	Whittaker function
J_ν, N_ν	Bessel functions of first and second kind
I_ν, K_ν	Modified Bessel functions of first and second kind
V_1, I_1	Voltage and current (under s.s. sinusoidal variation) at sending end
V_2, I_2	Voltage and current (under s.s. sinusoidal variation) at receiving end
N	Two-port network
$\underline{Z}$	The matrix of Z-parameters
$\underline{Y}$	The matrix of Y-parameters
$\underline{a}$	The matrix of a-parameters
$\underline{g}$	The matrix of g-parameters
$\underline{h}$	The matrix of h-parameters

1.1 Background

For the past several years efforts have been made to reduce the physical size of electronic devices and considerable progress has been made so far. This attempt at miniaturization has given a great deal of impetus to areas such as micro-electronics, thin film circuitry, etc. One of the widely used miniature devices is the distributed R-C structure.

One of the applications of these structures is the null-network (notch filter). In 1960 Kaufman [5] introduced a notch network, constructed from a basic distributed R-C structure and one lumped resistor, which produces sharper nulls than the conventional R-C notch network realized with lumped elements.

Another application of the R-C distributed structure is as a phase shift oscillator. This has been analyzed by Edson [6]. Here again it is found that the oscillator with distributed parameters has a better performance characteristic than the lumped parameter oscillator, in the sense that the gain requirement is lower and the stability is improved.

Hesselberth [7] has shown that certain configurations of the distributed R-C structures can be used as a high pass filter, although the characteristic of the R-C structure is similar to that of a low pass filter.

The distributed R-C structure also finds application in digital computer logic circuits such as the "NOR" speedup network [8].

It has been shown by Kaufman and Garrett [9] that the mathematical model of a distributed R-C structure is a second order partial differential

equation with variable coefficients. Using the notation in Figure 1.1, the resulting equation for the voltage $v(x,t)$ is given by,

$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{r(x)} \cdot \frac{dr(x)}{dx} \cdot \frac{\partial v}{\partial x} = r(x) \cdot c(x) \cdot \frac{\partial v}{\partial t} \quad \dots (1.1)$$

The derivation of this equation is given in Appendix 1. In a similar way the equation for the current $i(x,t)$ can be written as,

$$\frac{\partial^2 i}{\partial x^2} - \frac{1}{c(x)} \cdot \frac{dc(x)}{dx} \cdot \frac{\partial i}{\partial x} = r(x) \cdot c(x) \cdot \frac{\partial i}{\partial t} \quad \dots (1.2)$$

Equations (1.1) and (1.2) can be solved only for certain specified types of coefficients. This then means that an exact analysis of the distributed R-C structure is possible for only certain types of nonuniformity.

One approach that has been used by some researchers is to assume that the product of $r(x)$ and $c(x)$ is invariant. The solutions of equations (1.1) and (1.2) are easily obtained if this assumption is made. Su [10,11] has considered cases where $r(x)$ and/or $c(x)$ are trigonometric and/or hyperbolic functions of the space variable x . Kaufman and Garrett [9] have considered exponential variation of $r(x)$ and $c(x)$.

In a recent publication Gough and Gould [1] have used an indirect approach to this problem. The results of Gough and Gould are reproduced below for the R-C case.

Let,

$$\mathcal{L}[v(x,t)] = V(x,s) \quad \dots (1.3)$$

$$\mathcal{L}\left[\frac{\partial}{\partial t} v(x,t)\right] = sV(x,s) \quad \dots (1.4)$$

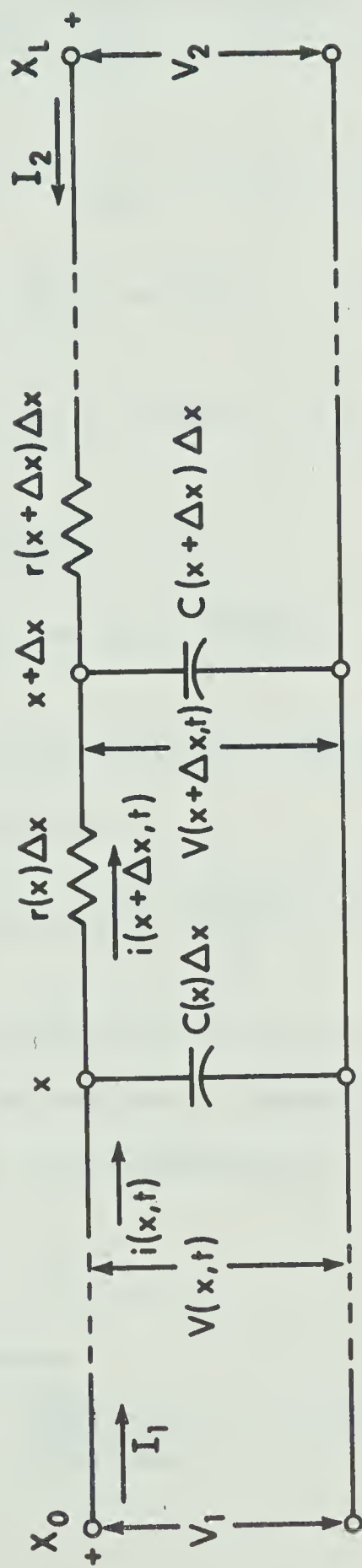


FIG 1.1 EQUIVALENT CIRCUIT OF INCREMENTAL LENGTH Δx OF R - C STRUCTURE

where $\mathcal{L}$ is the Laplace Transformation with respect to the variable t .

Taking the Laplace Transformation of both sides of equation (1.1), we get,

$$\frac{\partial^2 V(x,s)}{\partial x^2} - \frac{1}{r(x)} \cdot \frac{dr(x)}{dx} \cdot \frac{\partial V(x,s)}{\partial x} = r(x) \cdot c(x) \cdot s \cdot V(x,s) \quad \dots (1.5)$$

we can write,

$$\left. \begin{aligned} r(x) &= Z(x), & \text{and} & & c(x) &= Y(x), \\ \text{and} \quad \frac{1}{r(x)} \cdot \frac{dr(x)}{dx} &= \frac{d}{dx} \cdot \log Z(x) \end{aligned} \right\} \quad \dots (1.6)$$

This notation is being used to conform to the published work of Gough and Gould.

Then equation (1.5) becomes,

$$\frac{\partial^2 V(x,s)}{\partial x^2} - \frac{d}{dx} [\log Z(x)] \frac{\partial V(x,s)}{\partial x} = s \cdot Z(x) \cdot Y(x) \cdot V(x,s) \quad \dots (1.7)$$

For sinusoidal variation of $V(x,t)$ with respect to t , equation (1.7) becomes under steady state conditions,

$$\frac{d^2 V(x)}{dx^2} - \frac{d}{dx} [\log Z(x)] \cdot \frac{dV(x)}{dx} - k \cdot Z(x) \cdot Y(x) \cdot V(x) = 0 \quad \dots (1.8)$$

where k is the frequency factor ($k = j\omega$) and $\log$ is the natural logarithm.

The independent variable x (physical length) is transformed to an electrical length z , by the transformation,

$$z = \int_0^x \sqrt{\frac{Z}{Y}} \cdot dx \quad \dots (1.9)$$

then equation (1.8) becomes,

$$\frac{d^2 V}{dz^2} - \frac{d}{dz} \left(\log \sqrt{\frac{Z(z)}{Y(z)}} \right) \frac{dV}{dz} - k \cdot V = 0 \quad \dots (1.10)$$

where $Z(z) = Z(x) \Big|_{x=f(z)}$, and the relationship between x and z is given by equation (1.9). This type of notation will be used throughout this thesis. By means of another transformation,

$$V(z) = U(z) \cdot \left(\frac{Z}{Y}\right)^{\frac{1}{4}} \quad \dots (1.11)$$

equation (1.10) is reduced to the form,

$$\frac{d^2 U}{dz^2} + \left[\frac{1}{2} \cdot \frac{d^2}{dz^2} \left(\log \sqrt{\frac{Z}{Y}} \right) - \frac{1}{4} \cdot \left(\frac{d}{dz} \log \sqrt{\frac{Z}{Y}} \right)^2 - k \right] U = 0 \quad \dots (1.12)$$

which can be written as,

$$\frac{d^2 U}{dz^2} + [F(z) - k] \cdot U = 0 \quad \dots (1.13)$$

where

$$F(z) = \frac{1}{2} \cdot \frac{d^2}{dz^2} \left(\log \sqrt{\frac{Z}{Y}} \right) - \left[\frac{1}{4} \left(\frac{d}{dz} \log \sqrt{\frac{Z}{Y}} \right)^2 \right] \quad \dots (1.14)$$

$F(z)$ is referred to as the intermediate function in [1]. Equation (1.14) is in the Riccati Form but is changed into a second order linear equation in the normal form by the transformation,

$$\Phi = \left(\frac{Z}{Y}\right)^{-\frac{1}{4}} \quad \dots (1.15)$$

yielding

$$\frac{d^2 \Phi}{dz^2} + F(z) \cdot \Phi = 0 \quad \dots (1.16)$$

then

$$V(z) = U(z) \cdot \Phi^{-1}(z) \quad \dots (1.17)$$

An infinite number of combinations of $Z(x)$ and $Y(x)$ which satisfies a given $F(z)$ can be found. For each such pair of $Z(x)$ and $Y(x)$, the voltage $V(x, \omega)$ can be found if the boundary conditions are known. For instance, if $F(z) = \text{constant}$, we get exponential solutions for equations (1.13) and (1.16); for other results see [1]. It should be noted that the crucial step in this method is the transformation of the dependent variable $V(x)$.

1.2 Objectives of This Study

In this thesis, a different method will be used. This method can be considered as a variation of the method of Gough and Gould. Only the independent variable x will be transformed. The dependent variable $V(x)$ will not be transformed. It will be seen that this transformation has some advantages over the method of Gough and Gould. This will be explained later in this thesis. Expressions for the electrical characteristics such as the Z , Y , and h parameters of the distributed R-C structure will also be derived. The electrical equivalence, which will be defined later in this thesis, of two or more R-C structures with different dimensions and distributions will be discussed.

2.1 General Solution of the Differential Equation

We shall start with the basic equation (1.8),

$$\frac{d^2V}{dx^2} - \left(\frac{Z'}{Z}\right) \cdot \frac{dV}{dx} - k \cdot Z \cdot Y \cdot V = 0 \quad \dots (2.1)$$

which can be put in another form as,

$$\frac{d^2V}{dx^2} - \left(\frac{d}{dx} \log Z\right) \cdot \frac{dV}{dx} - k \cdot Z \cdot Y \cdot V = 0 \quad \dots (2.2)$$

$Z(x)$ and $Y(x)$ are differentiable in the closed interval $[0,x]$. If they are undefined at $x = 0$, this point is excluded.

Let us make the following transformation of the independent variable x ,

$$y = \int_0^x Z(x) \cdot dx \quad \dots (2.3)$$

See Appendix (2) for the significance of this transformation. This transformation reduces equation (2.2) to the form,

$$\frac{d^2V}{dy^2} - k\left(\frac{Y}{Z}\right)V = 0 \quad \dots (2.4)$$

The solution of equation (2.4) is made use of in Chapter 3, where equivalence of two R-C structures which have the same $\frac{Y}{Z}$ ratio with respect to y is discussed. It is worth noting that while the entire discussion assumes that the elements R and C are ideal, the case of a lossy dielectric can be easily handled by replacing k in equation (2.1) by $k + \frac{G}{Y}$, where G is the shunt conductance per unit length. This of course assumes both G and Y to have the same variation with respect to the variable x .

We shall now make a further transformation,

$$\theta = \sqrt{k} \cdot y \quad \dots (2.5)$$

in order to eliminate k from equation (2.4). This gives,

$$\frac{d^2V}{d\theta^2} - \left(\frac{Y}{Z}\right) \cdot V = 0 \quad \dots (2.6)$$

The two transformations in equation (2.3) and (2.5) can be combined, if necessary. The combined transformation is,

$$\theta = \int_0^x \sqrt{k} \cdot Z \cdot dx \quad \dots (2.7)$$

The general solution of equation (2.6) is,

$$V(\theta) = k_1 p(\theta) + k_2 q(\theta) \quad \dots (2.8)$$

Using the inverse transformation,

$$x = \int_0^\theta \frac{1}{\sqrt{k} \cdot Z} \cdot d\theta \quad \dots (2.9)$$

we get,

$$V(x) = k_1 \hat{p}(x) + k_2 \hat{q}(x) \quad \dots (2.10)$$

From the Laplace Transformation of equation (A1.3) with respect to the variable t , we get,

$$I(x,s) = - \frac{1}{Z(x)} \cdot \frac{\partial V(x,s)}{\partial x} \quad \dots (2.11)$$

which for sinusoidal steady state becomes,

$$I(x,k) = - \frac{1}{Z(x)} \cdot \frac{\partial V(x,k)}{\partial x} \quad \dots (2.12)$$

where $k = j\omega$.

Later on in this thesis, computations will be carried out at specific values of ω . Consequently, k can be considered to be a constant in equation (2.12), and the partial derivatives replaced by total derivatives with respect to x . Equation (2.12) becomes,

$$I(x) = - \frac{1}{Z(x)} \cdot \frac{dV(x)}{dx} \quad \dots (2.13)$$

Substituting for $V(x)$ from equation (2.10), we get,

$$I(x) = - \frac{1}{Z(x)} [k_1 \hat{p}'(x) + k_2 \hat{q}'(x)] \quad \dots (2.14)$$

where the values of the arbitrary constants k_1 and k_2 can be evaluated if the boundary conditions are specified.

We will conclude this section by discussing a few examples.

Example 2.1

Let $Z(\theta) = r_o$, a constant, and $Y(\theta) = c_o$, a constant. The ratio $\frac{Y(\theta)}{Z(\theta)} = c_o/r_o$, a constant. For this case the solution of equation (2.6) is,

$$V(\theta) = k_1 \exp \left(\sqrt{\frac{c_o}{r_o}} \theta \right) + k_2 \exp \left(-\sqrt{\frac{c_o}{r_o}} \theta \right) \quad \dots (2.15)$$

By using the inverse transformation,

$$x = \int_0^\theta \frac{1}{o\sqrt{k} \cdot r_o} \cdot d\theta = \frac{1}{\sqrt{k}} \cdot \frac{1}{r_o} \cdot \theta \quad \dots (2.16)$$

we get, from equation (2.15),

$$V(x) = k_1 \exp(\sqrt{kr_0c_0} \cdot x) + k_2 \exp(-\sqrt{kr_0c_0} \cdot x) \quad \dots (2.17)$$

which is the well known solution of the uniform R-C structure.

Example 2.2

Let

$$Z(\theta) = r_0 \left(1 + \frac{m}{r_0} \cdot \frac{\theta}{\sqrt{k}} \right) \quad \dots (2.18)$$

and

$$Y(\theta) = c_0 \frac{1}{\left(1 + \frac{m}{r_0} \cdot \frac{\theta}{\sqrt{k}} \right)} \quad \dots (2.19)$$

or

$$\frac{Y(\theta)}{Z(\theta)} = \frac{c_0}{r_0} \cdot \left(1 + \frac{m}{r_0} \cdot \frac{\theta}{\sqrt{k}} \right)^{-2} \quad \dots (2.20)$$

This will lead to the well known solution of the exponential R-C structure [9].

$$V(x) = \exp\left(\frac{m}{2} \cdot x\right) \left\{ k_1 \cosh \left[\left(\frac{m}{2} \sqrt{1 + \frac{4kr_0c_0}{m^2}}\right)x \right] + k_2 \sinh \left[\left(\frac{m}{2} \sqrt{1 + \frac{4kr_0c_0}{m^2}}\right)x \right] \right\} \quad \dots (2.21)$$

A number of other distributions has been considered. These cases are tabulated in Table 2.1.

2.2 Detailed Solutions for Some R-C Structures

In this section we will proceed along the same lines as in the previous section, with one difference. The solution of the differential equation (2.6) for the R-C structure with two specified $\frac{Y}{Z}$ ratios will be

$Y(\theta)/Z(\theta)$	$V(\theta)$	REMARKS
1) $\alpha + (\beta/\theta) + (\gamma/\theta^2)$ WHERE α, β AND γ ARE CONSTANTS (REAL OR COMPLEX)	$W_{-(\beta/\zeta), m}(\zeta \theta)$ $\zeta^2 = 4\alpha$ $4m^2 = 1 + 4\gamma$	WHITTAKER FUCTION (12)
2) $\alpha - b^2 \theta^2$ WHERE α AND b ARE CONSTANTS (REAL OR COMPLEX)	$\theta^{-1/2} W_{\frac{ia}{4b}, \frac{1}{4}}(ib \theta^2)$	"
3) $\alpha \theta^{q-2} - b^2 \theta^{2q-2}$ WHERE α AND b ARE COMPLEX CONSTANTS AND q IS A REAL CONSTANT	$\theta^{(1-q)/2} W_{ia/2bq, 1/2q}(i2b/q \theta^q)$	"
4) $c \theta^{2q-2}$ WHERE c IS A COMPLEX CONSTANT, q IS A REAL CONSTANT	$\sqrt{\theta} Z_{1/2q}(i\sqrt{c} \theta^q/q)$	BESSEL FUCTION (12,13)
5) $b^2(a^2 e^{2b\theta} - \nu^2)$ WHERE a AND b ARE COMPLEX CONSTANTS AND ν IS A REAL CONSTANT	$Z_\nu[a \exp(ib\theta)]$	"
6) $(a+b\theta)^Q$ WHERE a AND b ARE COMPLEX CONSTANTS AND Q IS REAL	$\sqrt{a+b\theta} Z_{\frac{1}{Q+2}} \left[\frac{i \frac{b}{Q+2}}{(a+b\theta)^{\frac{Q+2}{2}}} \right]$	"

TABLE 2.1 SOLUTION OF EQUATION (2.6) FOR DIFFERENT $[Y(\theta)/Z(\theta)]$ RATIOS

carried out in some detail.

The results of Example 2.4 will be used in the next section to obtain the frequency response of the particular R-C structure.

Example 2.3

Let us assume

$$Z(\theta) = r_o \cdot \left(\frac{\alpha + 1}{r_o \sqrt{k}} \right)^{\frac{\alpha}{\alpha+1}} \cdot \theta^{\frac{\alpha}{\alpha+1}} \quad \dots (2.22)$$

and

$$Y(\theta) = c_o \cdot \left(\frac{\alpha + 1}{r_o \sqrt{k}} \right)^{\frac{\beta}{\alpha+1}} \cdot \theta^{\frac{\beta}{\alpha+1}} \quad \dots (2.23)$$

we have,

$$\frac{Y(\theta)}{Z(\theta)} = \frac{c_o}{r_o} \cdot \left(\frac{\alpha + 1}{r_o \sqrt{k}} \right)^{(\beta-\alpha)/(\alpha+1)} \cdot \theta^{(\beta-\alpha)/(\alpha+1)} \quad \dots (2.24)$$

By using the inverse transform,

$$x = \int_0^\theta \frac{1}{\sqrt{k} Z(\theta)} \cdot d\theta \quad [2.9]$$

we get for the present choice of $Z(\theta)$;

$$x = \left(\frac{\alpha + 1}{r_o \sqrt{k}} \right)^{\frac{1}{\alpha+1}} \cdot \theta^{\frac{1}{\alpha+1}} \quad \dots (2.25)$$

If we solve equation (2.25) for θ we get,

$$\theta = \left(\frac{r_o \sqrt{k}}{\alpha + 1} \right) \cdot x^{\alpha+1} \quad \dots (2.26)$$

Substituting this result into equation (2.22) and (2.23) we get after simplification,

$$\left. \begin{aligned} Z(x) &= r_0 \cdot x^\alpha \\ Y(x) &= c_0 \cdot x^\beta \end{aligned} \right\} \dots (2.27)$$

Let us define some relations:

$$\nu = (\alpha + 1)/(\alpha + \beta + 2) \dots (2.28)$$

$$\gamma = (\alpha + \beta + 2)/[2(\alpha + 1)] \dots (2.29)$$

$$q = 2/(\alpha + \beta + 2) \dots (2.30)$$

$$b = j \cdot \sqrt{\frac{c_0}{r_0}} \cdot \left(\frac{\alpha + 1}{r_0 \sqrt{k}}\right)^{(\beta - \alpha)/[2(\alpha + 1)]} \dots (2.31)$$

The solution of equation (2.6) with (Y/Z) ratio as given by equation (2.24) [see item (4), Table 2.1], is

$$V(\theta) = (\theta)^{\frac{1}{2}} \cdot Z_\nu[b(\theta^\gamma/\gamma)] \dots (2.32)$$

where Z_ν is a linear combination of Bessel functions of order ν . Substituting for θ from equation (2.26), equation (2.32) becomes,

$$V(x) = x^{\nu/q} Z_\nu\left[j \cdot \frac{2\sqrt{kr_0 c_0}}{(\alpha + \beta + 2)} x^{1/q}\right] \dots (2.33)$$

All arbitrary constants are included in Z_ν . This result agrees with the result given in McLachlan's book [14] using a different approach.

Equation (2.33) can be put in another form,

$$V(x) = x^{\nu/q} \left[A J_\nu\left(j \frac{2\sqrt{kr_0 c_0}}{\alpha + \beta + 2} \cdot x^{1/q}\right) + B N_\nu\left(j \frac{2\sqrt{kr_0 c_0}}{\alpha + \beta + 2} \cdot x^{1/q}\right) \right] \dots (2.34)$$

where J_ν and N_ν are Bessel functions of first and second kind respectively of

order ν .

Equation (2.34) can be put in another form,

$$V(x) = x^{\nu/q} \left[A I_{\nu} \left(\frac{2\sqrt{kr_0 c_0}}{\alpha + \beta + 2} \cdot x^{1/q} \right) + B k_{\nu} \left(\frac{2\sqrt{kr_0 c_0}}{\alpha + \beta + 2} \cdot x^{1/q} \right) \right] \dots (2.35)$$

where I_{ν} and k_{ν} are the modified Bessel functions of first and second kind respectively. The usefulness of equation (2.35) will be evident from Table 2.2, where the expressions for $V(x)$ for specific values of α and β are tabulated. It is seen that the values of α and β chosen correspond in several cases to commonly encountered distributions of Z and Y .

Example 2.4

In this example the ratio (Y/Z) is taken to be,

$$\frac{Y(\theta)}{Z(\theta)} = \left(a + \frac{b}{\sqrt{k}} \theta \right)^Q \dots (2.36)$$

From item 6, Table 2.1, the corresponding solution for the voltage as a function of θ is given by,

$$V(\theta) = \left(a + \frac{b}{\sqrt{k}} \theta \right)^{\frac{1}{2}} \cdot Z_{\frac{1}{Q+2}} \left[j \frac{\sqrt{k}}{b} \cdot \frac{\left(a + \frac{b}{\sqrt{k}} \theta \right)^{(Q+2)/2}}{(Q+2)/2} \right] \dots (2.37)$$

where $Z_{\frac{1}{Q+2}}$ is a linear combination of Bessel functions of order $1/(Q+2)$.

Let

$$Z(\theta) = \text{unity}. \dots (2.38)$$

From equation (2.9) we get,

$$\theta = \sqrt{k} \cdot x \dots (2.39)$$

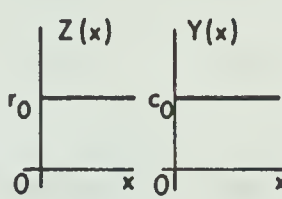
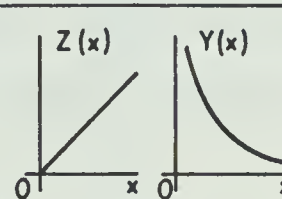
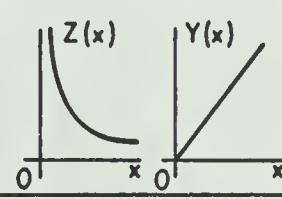
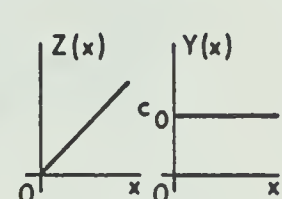
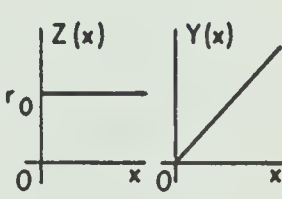
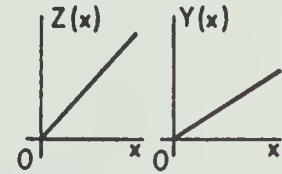
				SOLUTION		REMARKS
α	β	ν	q			
0	0	1/2	1	$V(x) = X^{1/2} \left[A I_{1/2} (\sqrt{k r_0 c_0} X) + B K_{1/2} (\sqrt{k r_0 c_0} X) \right]$ OR $A_1 \sinh (\sqrt{k r_0 c_0} X) + B_1 \cosh (\sqrt{k r_0 c_0} X)$		UNIFORM LINE
1	-1	1	1	$V(x) = X \left[A I_1 (\sqrt{k r_0 c_0} X) + B K_1 (\sqrt{k r_0 c_0} X) \right]$		
-1	1	0	1	$V(x) = A I_0 (\sqrt{k r_0 c_0} X) + B K_0 (\sqrt{k r_0 c_0} X)$		
1	0	2/3	2/3	$V(x) = X \left[A I_{2/3} (2/3 \sqrt{k r_0 c_0} X^{3/2}) + B K_{2/3} (2/3 \sqrt{k r_0 c_0} X^{3/2}) \right]$		
0	1	1/3	2/3	$V(x) = X^{1/2} \left[A I_{1/3} (2/3 \sqrt{k r_0 c_0} X^{3/2}) + B K_{1/3} (2/3 \sqrt{k r_0 c_0} X^{3/2}) \right]$		
1	1	1/2	1/2	$V(x) = X \left[A I_{1/2} (1/2 \sqrt{k r_0 c_0} X^2) + B K_{1/2} (1/2 \sqrt{k r_0 c_0} X^2) \right]$		

TABLE 2.2 EXPRESSIONS FOR THE VOLTAGE $V(x)$ FOR DIFFERENT DISTRIBUTION (EXAMPLE 2.3)

Substituting in equation (2.37) we get,

$$V(x) = (a + bx)^{\frac{1}{2}} \cdot Z \frac{1}{Q+2} \left[j \frac{\sqrt{k}}{b} \cdot \frac{(a + bx)^{(Q+2)/2}}{(Q + 2)/2} \right] \dots (2.40)$$

Table (2.3) gives the solutions for specific values of Q.

2.3 The Frequency Response of the Distributed R-C Structure

In this section the solutions derived earlier will be used to obtain the frequency response of the R-C structure. It is well known that the open circuit voltage transfer function of a two-port network N (Fig. 2.1), under sinusoidal steady state conditions, is

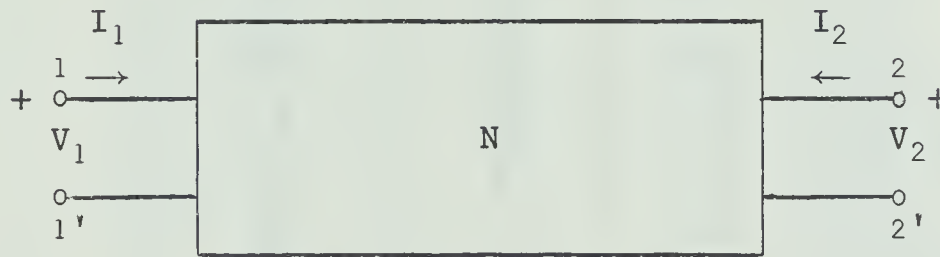


Fig. 2.1 Two-Port Network

$$\left. \frac{V_2}{V_1} \right|_{I_2 = 0} = \frac{Z_{21}}{Z_{11}} \dots (2.41)$$

where Z_{11} is the driving point impedance at port 11',

$$\text{i.e., } Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2 = 0} \dots (2.42)$$

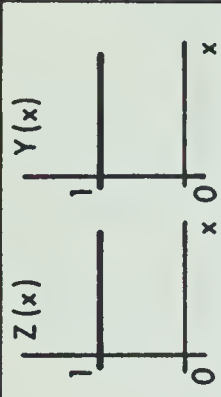
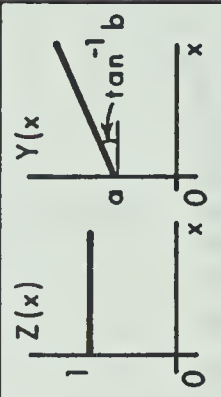
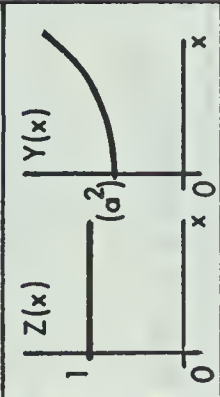
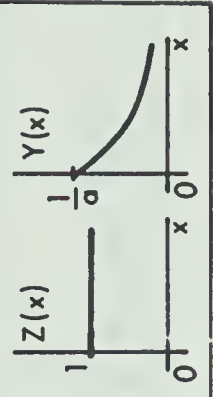
	Q	SOLUTION	
	0	$V(x) = A \sinh(\sqrt{k} \cdot x) + B \cosh(\sqrt{k} \cdot x)$	
	1	$V(x) = (a + bx)^{1/2} \left[AI_{1/3} \left(\sqrt{k/b} x (a + bx)^{3/2} / (3/2) \right) + BK_{1/3} \left(\sqrt{k/b} x (a + bx)^{3/2} / (3/2) \right) \right]$	
	2	$V(x) = (a + bx)^{1/2} \left[AI_{1/4} \left(\sqrt{k/b} x (a + bx)^2 / 2 \right) + BK_{1/4} \left(\sqrt{k/b} x (a + bx)^2 / 2 \right) \right]$	
	-1	$V(x) = (a + bx)^{1/2} \left[AI_1 \left(\sqrt{k/b} x (a + bx)^{1/2} / (1/2) \right) + BK_1 \left(\sqrt{k/b} x (a + bx)^{1/2} / (1/2) \right) \right]$	

TABLE 2.3 EXPRESSIONS FOR THE VOLTAGES $V(x)$ FOR DIFFERENT DISTRIBUTIONS (EXAMPLE 2.4)

	Q	SOLUTION
1	-4/3	$V(x) = (a + bx)^{1/2} \left[AI_{3/2}(\xi) + BK_{3/2}(\xi) \right]$ $\xi = 3 \sqrt{k/b} \times (a + bx)^{1/3}$
2	-8/5	$V(x) = (a + bx)^{1/2} \left[AI_{5/2}(\xi) + BK_{5/2}(\xi) \right]$ $\xi = 5 \sqrt{k/b} \times (a + bx)^{1/5}$
3	-12/7	$V(x) = (a + bx)^{1/2} \left[AI_{7/2}(\xi) + BK_{7/2}(\xi) \right]$ $\xi = 7 \sqrt{k/b} \times (a + bx)^{1/7}$

TABLE 2.3 CONTINUED

and Z_{21} is the transfer impedance between ports 11' and 22',

$$\text{i.e.,} \quad Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2 = 0} \quad \dots (2.43)$$

Dutta Roy [15] has obtained expressions for the $\underline{Z}$, $\underline{Y}$, $\underline{g}$, $\underline{a}$, and $\underline{h}$ parameters for a distributed R-C structure. These are summarized in Appendix 3. The expressions for $\underline{Z}$ will be used in this section.

A specific example will now be considered, namely, the R-C structure discussed in example 2.4. Equation (2.40) which gives the solution of voltage along the structure can be put in the form,

$$V(x) = (a + bx)^{\frac{1}{2}} Z_v(\eta) \quad \dots (2.44)$$

From the basic equation (A1.3) (keeping in mind that $Z(x) = r(x) = 1$), and the following property of Bessel functions,

$$\frac{dZ_v}{dx} = \frac{1}{2}(Z_{v-1} - Z_{v+1}) \quad \dots (2.45)$$

we get the following expression for the current along the structure,

$$I(x) = - \{ (a+bx)^{\frac{1}{2}} \cdot \frac{1}{2}[Z_{v-1}(\eta) - Z_{v+1}(\eta)] \cdot \eta' + b(a+bx)^{-\frac{1}{2}} \cdot \frac{1}{2}[Z_v(\eta)] \} \quad \dots (2.46)$$

Since the order of the Bessel function depends on Q , the expressions for $V(x)$ and $I(x)$ depend on Q .

Following the steps explained in Appendix 3 and using equations (2.41), (2.44), and (2.46), we can get the frequency response of this R-C

structure for different values of a , b , and Q . The frequency response curves for a number of different distributions are plotted in Figures 2.2 through 2.6. The necessary computations were carried out on IBM 360 digital computer using Fortran IV language. A program for computing Bessel functions with complex arguments and orders was available only in Fortran II language. This program was modified by the writer using Fortran IV language. The tables of the relevant computer results are given in Appendix 4.

The following points are worth noting:

- 1) The gain tends to decrease and the entire frequency response curves to become more flat as Q is increased.
- 2) The curve corresponding to $Q = -1.0$ has a sharper cut-off than the rest. Such a distribution can be used effectively in a low pass filter.
- 3) For positive values of Q , the gain decreases as " a " increases (" a^Q " is the capacitance at $x = 0$).

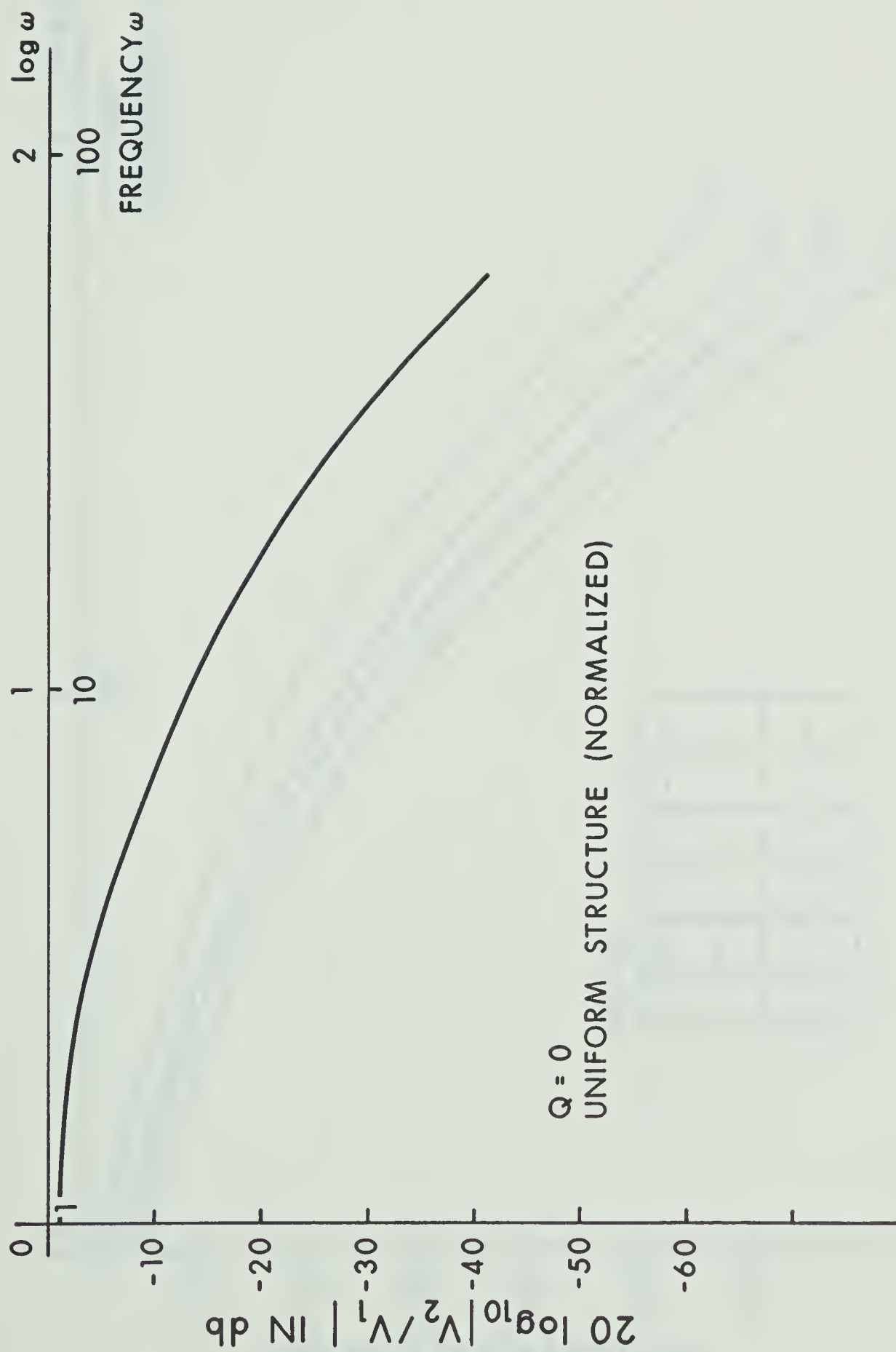


FIG 2.2 FREQUENCY RESPONSE OF UNIFORM R-C STRUCTURE,
($Z(x) = 1$, $Y(x) = 1$)

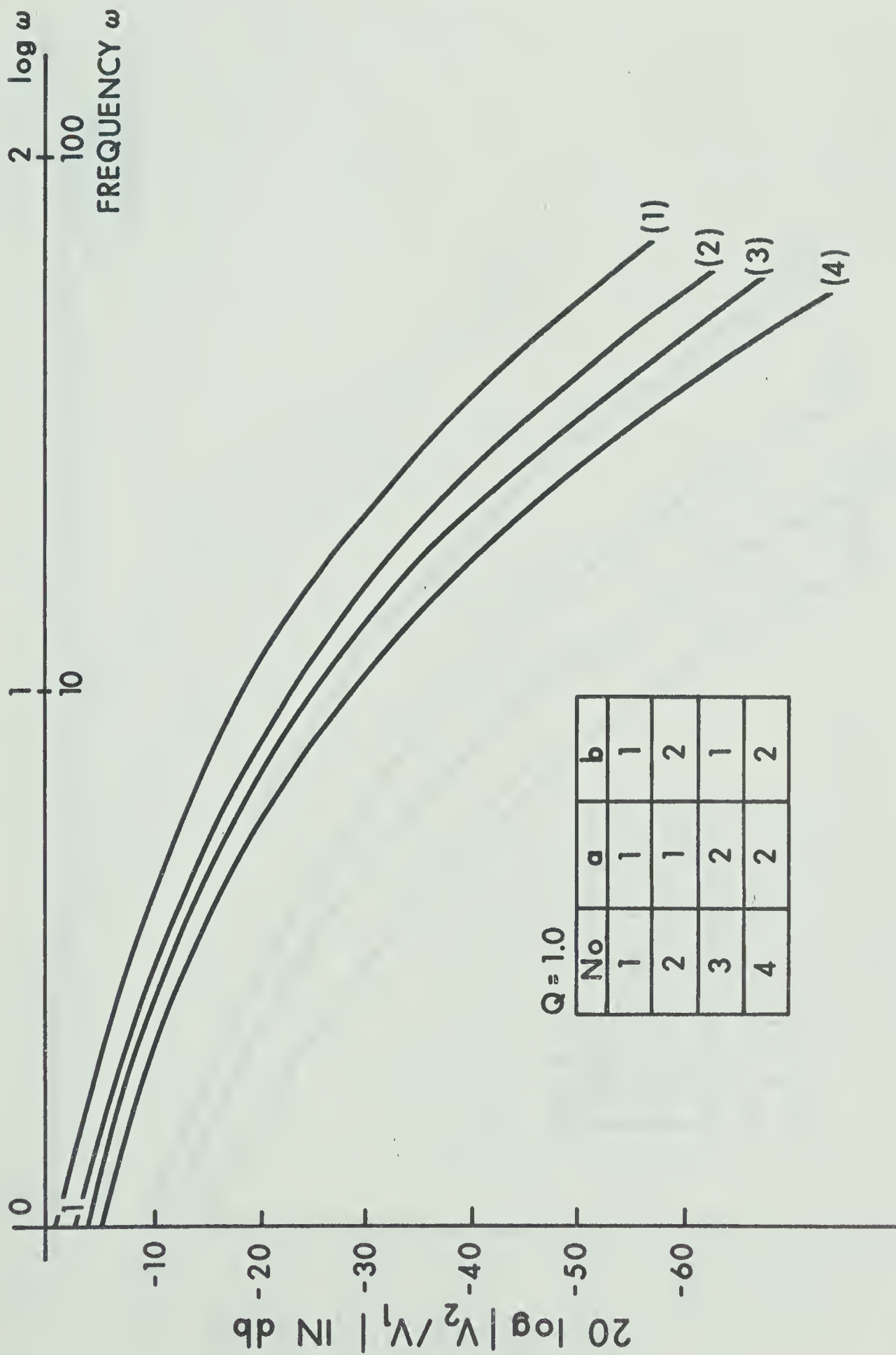


FIG. 2.3 FREQUENCY RESPONSE OF R-C STRUCTURE (UNIFORM $Z(x)$, AND LINEAR VARIATION OF $Y(x)$)

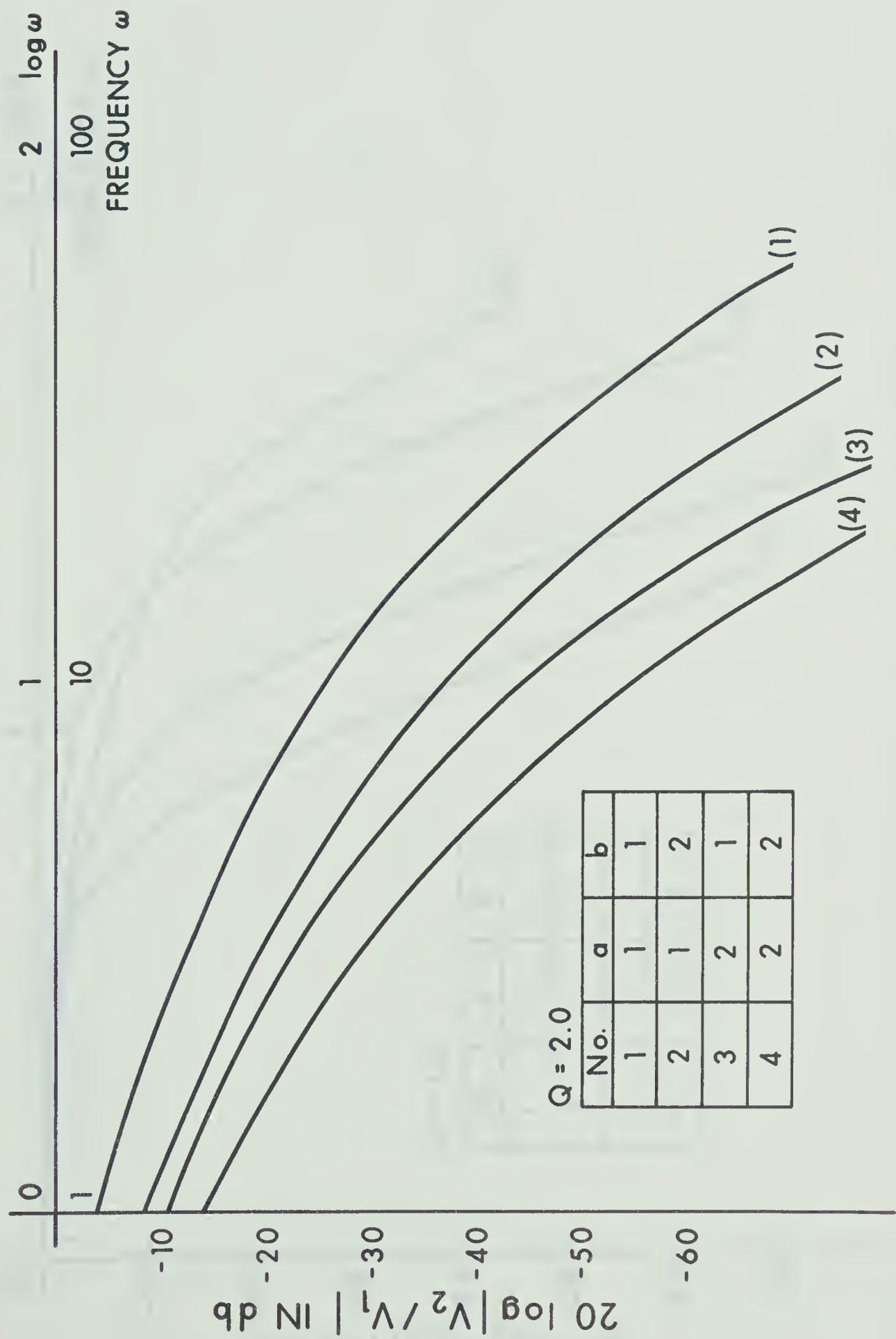


FIG. 2.4 FREQUENCY RESPONSE OF R-C STRUCTURE (UNIFORM $Z(x)$, AND PARABOLIC VARIATION IN $Y(x)$)

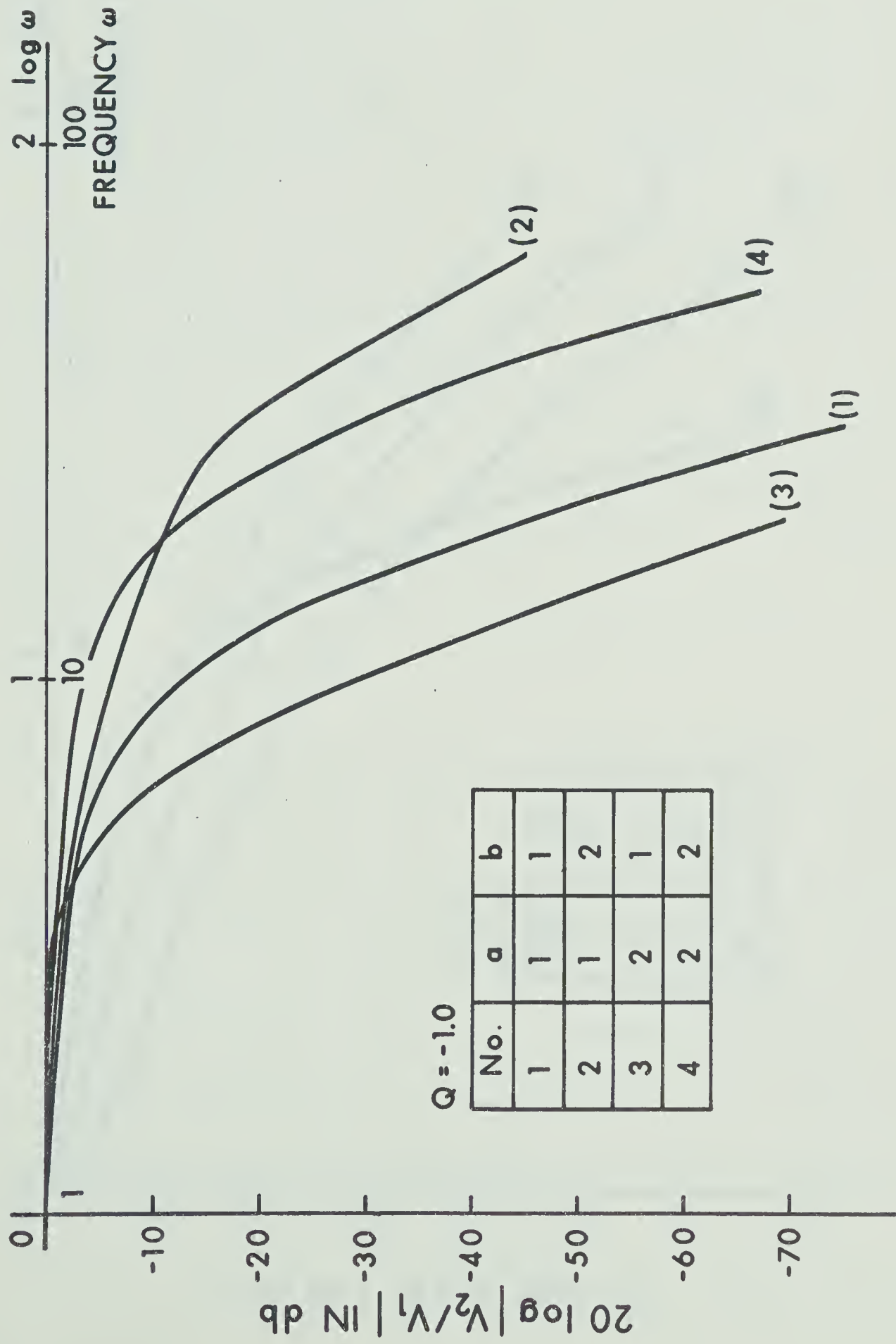


FIG. 2.5 FREQUENCY RESPONSE OF R-C STRUCTURE, (UNIFORM $Z(x)$ AND HYPERBOLIC VARIATION OF $Y(x)$)

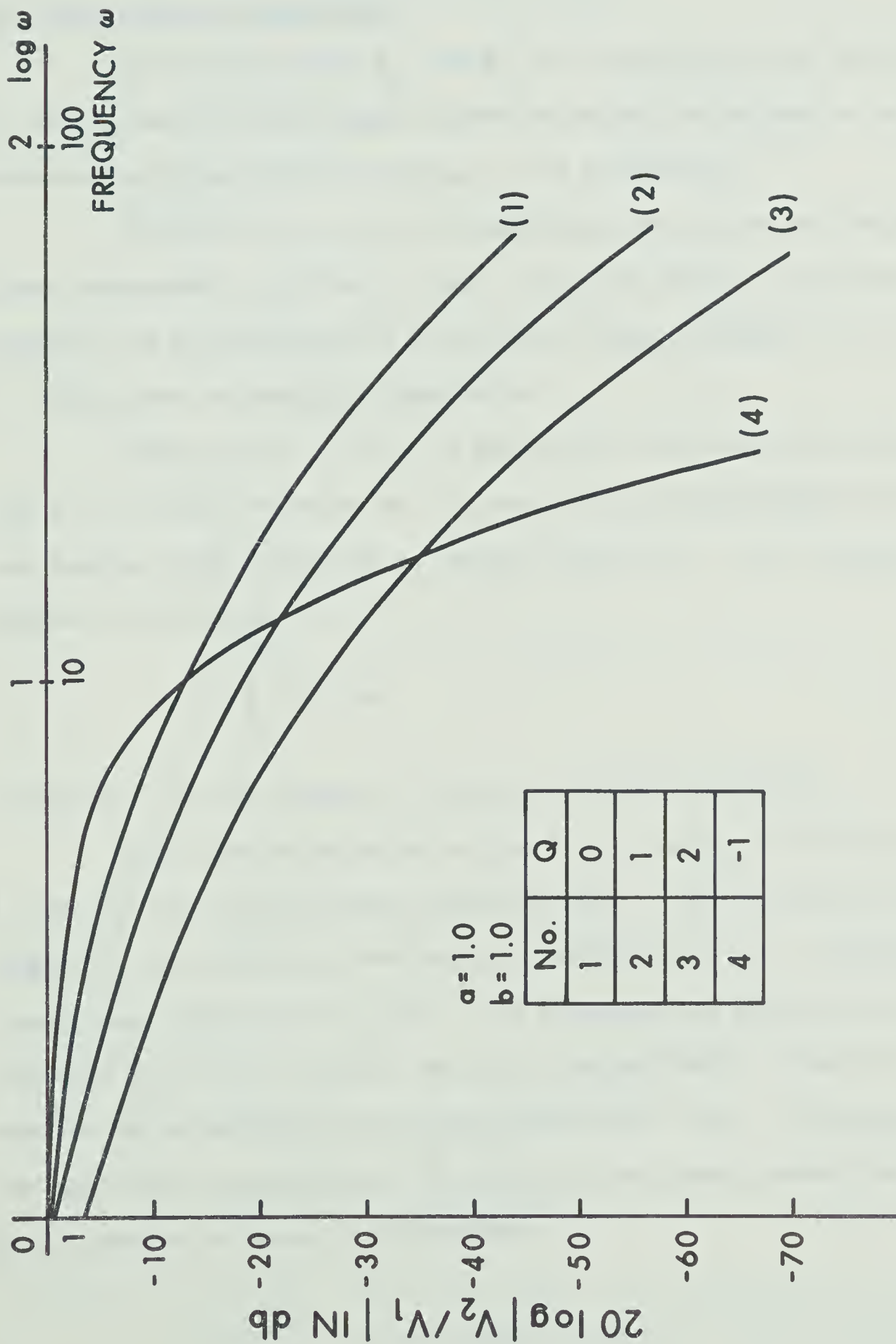


FIG 2.6 EFFECT OF CHANGE IN THE DISTRIBUTION OF Y ON THE FREQUENCY RESPONSE

3.1 The Concept of Equivalence

Two R-C structures N_A , and N_B , are considered to be equivalent if N_A , and N_B , may be interchanged without affecting the voltage or current associated with any element external to the structures.

If two R-C structures are equivalent they cannot be distinguished by any measurements performed at their terminals, since the measuring apparatus can be considered as an external element. However, the structures N_A , and N_B , may be dissimilar internally.

From equation (2.4) it is easily seen that structures with the same Y to Z ratio, expressed as a function of y, are equivalent at structure lengths which correspond to the same values of y. The variable y is related to the distance x by,

$$y = \int_0^x Z(x) dx \quad \dots (3.1)$$

In general, the true lengths of the structures are not equal.

This idea is explained in Fig. 3.1. Consider two structures N_A , and N_B , with the following characteristics. The structure N_A , has a length L_A , and impedance (resistance) distribution $Z_A(x)$, and admittance (capacitance) distribution $Y_A(x)$. The corresponding quantities for the structure N_B , are L_B , $Z_B(x)$, and $Y_B(x)$ respectively. It will be assumed that the two structures have the same $[Y(y)/Z(y)]$ ratio. Consequently they are equivalent to each other. The length of the common transformed structure N_A is taken to be unity for convenience.

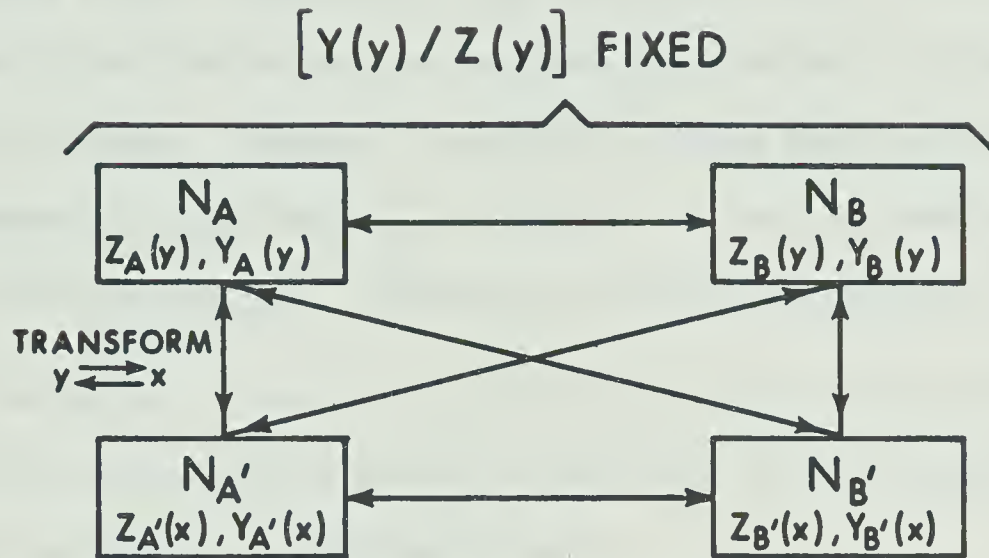


FIG. 3.1 EQUIVALENT R-C STRUCTURES

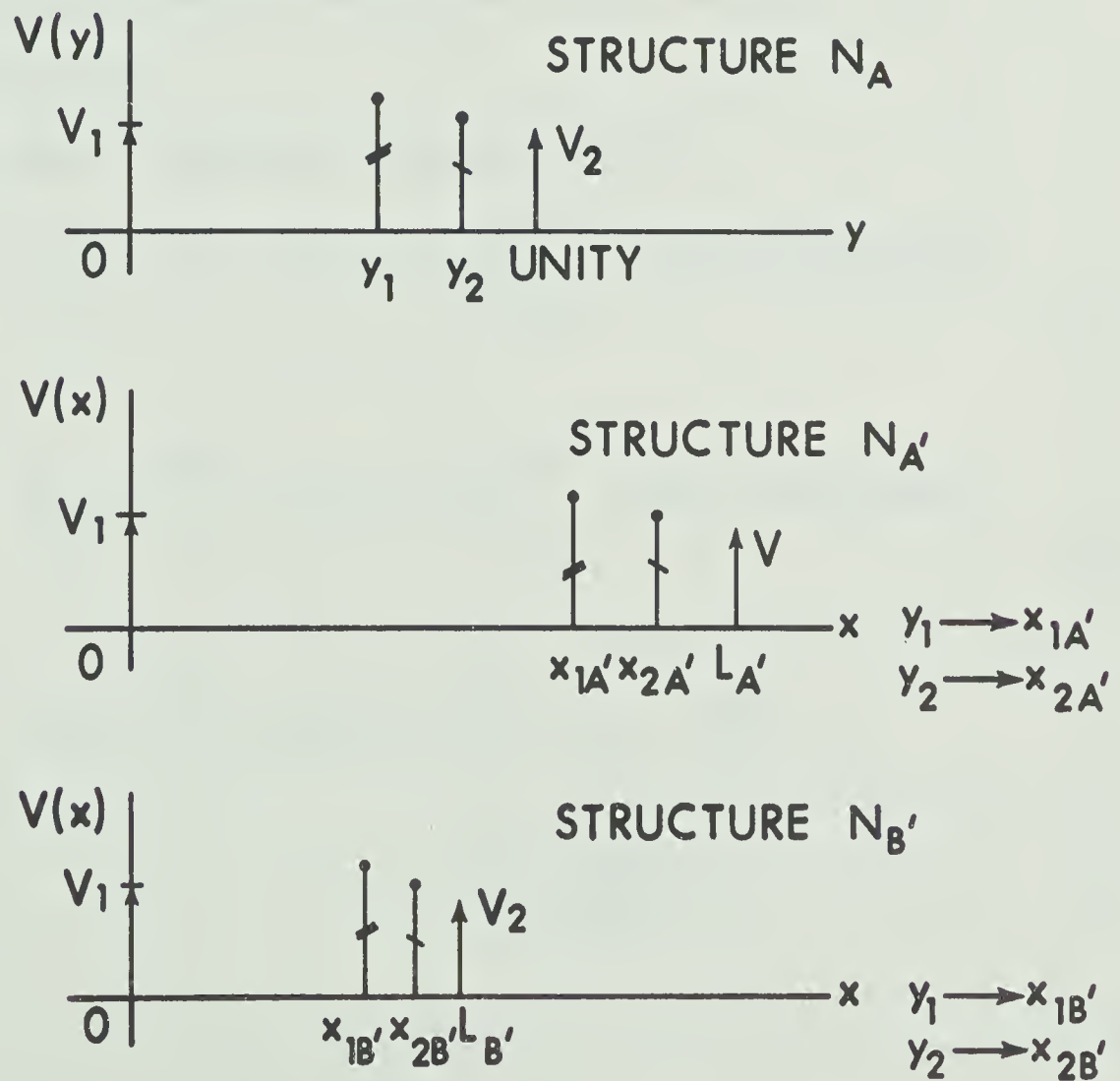


FIG. 3.2 VALUES OF VOLTAGES AT CORRESPONDING POINTS IN THE EQUIVALENT STRUCTURES

Although the lengths of $N_{A'}$, $N_{B'}$, and N_A are different in general, the boundary conditions, namely the values of the voltages and currents at the input and output terminals of the three structures, are the same. But the internal distribution of the voltages and currents in these structures may not be the same. However, it is worth noting that voltages (currents) at the points y_1 , $x_{1A'}$, and $x_{1B'}$ in Figure 3.2 are the same because $x_{1A'}$ and $x_{1B'}$ both map into y_1 . The same is true for the points y_2 , $x_{2A'}$, $x_{2B'}$. Or, in other words, if taps are placed on one line to form n-port, taps placed on the corresponding points of the other line as determined by $y = \int_0^x Z(x) dx$ make an equivalent n-port.

Next, it will be shown that the total impedance (resistance) of the equivalent structures $N_{A'}$ and $N_{B'}$ are equal, and that their total capacitances are equal.

The total resistance of $N_{A'}$ is given by,

$$R_{A'} = \int_0^{L_{A'}} Z_{A'}(x) dx = y \Big|_{y=0}^{y=1} = \text{unity (hypothesis)} \quad \dots (3.2)$$

and

$$R_{B'} = \int_0^{L_{B'}} Z_{B'}(x) dx = y \Big|_{y=0}^{y=1} = \text{unity (hypothesis)} \quad \dots (3.3)$$

Similarly,

$$\begin{aligned} C_{A'} &= \int_0^{L_{A'}} Y_{A'}(x) dx = \int_0^{L_A} Y_A(y) \cdot \frac{dx}{dy} \cdot dy \\ &= \int_0^{L_A} \left(\frac{Y_A(y)}{Z_A(y)} \right) dy \quad \dots (3.4) \end{aligned}$$

and

$$\begin{aligned} C_{B'} &= \int_0^{L_{B'}} Y_{B'}(x) dx = \int_0^{L_B} Y_B(y) \cdot \frac{dx}{dy} \cdot dy \\ &= \int_0^{L_B} \left(\frac{Y_B(y)}{Z_B(y)} \right) \cdot dy \end{aligned} \quad \dots (3.5)$$

But from hypothesis we have

$$L_A = L_B = \text{unity} \quad \dots (3.6)$$

and

$$\frac{Y_A(y)}{Z_A(y)} = \frac{Y_B(y)}{Z_B(y)} = \frac{Y(y)}{Z(y)} \quad \dots (3.7)$$

It follows that $C_{A'} = C_{B'}$.

The idea of equivalence which has been discussed so far in relation to two structures can be extended by considering the function $[Y(y)/Z(y)]$ as a generating function for several structures. This is shown graphically in Figure 3.3. For the sake of clarity the generation of equivalent structures has been shown for a particular function,

$$\frac{Y(y)}{Z(y)} = (a + b \cdot y)^Q \quad \dots (3.8)$$

which has been considered in section 2.2, example (2.4). This concept is applicable to other distributions as well.

The main advantage in generating several equivalent structures lies in the fact that some of the alternate structures may be more suitable for fabrication.

In the next section we will explore this concept of equivalence further by considering three examples.

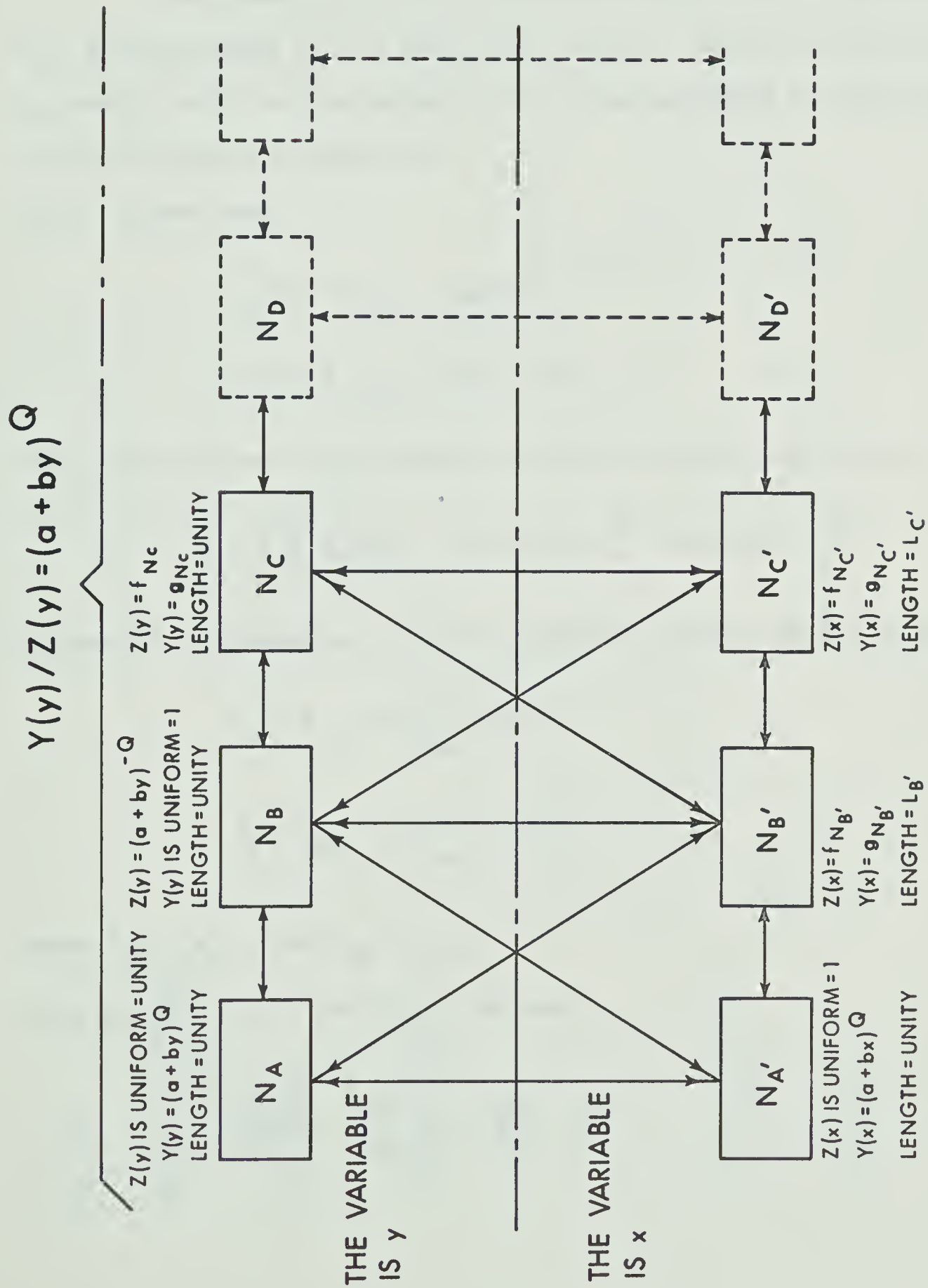


FIG. 3.3 GENERATION OF SEVERAL EQUIVALENT STRUCTURES

3.2 Some Examples

Example 3.1

This example will deal with the following case. Given a structure $N_{A'}$, with specified $Z_{A'}(x)$, and $Y_{A'}(x)$, and $L_{A'}$, obtain two other structures N_B , and N_C , which are equivalent to $N_{A'}$. The procedure is shown graphically in the flow chart in Figure 3.4.

Let us assume that,

$$Z_{A'}(x) = R_{A'O} \cdot \exp(m \cdot x) \quad \dots (3.9)$$

$$Y_{A'}(x) = C_{A'O} \cdot \exp(-m \cdot x) \quad \dots (3.10)$$

The relation between the variable y and the variable x is given by,

$$y = \int_0^x Z_{A'}(x) \cdot dx = R_{A'O} \cdot \frac{1}{m} \cdot [\exp(m \cdot x) - 1] \quad \dots (3.11)$$

By substituting equation (3.11) in equations (3.9) and (3.10) we get,

$$Z_A = R_{AO} \left(1 + \frac{m}{R_{AO}} \cdot y\right) \quad \dots (3.12)$$

$$Y_A = C_{AO} \left(1 + \frac{m}{R_{AO}} \cdot y\right)^{-1} \quad \dots (3.13)$$

where $R_{AO} = R_{A'O}$ and $C_{AO} = C_{A'O}$.

From equation (3.12) and (3.13) we have,

$$\frac{Z_A(y)}{Y_A(y)} = \frac{R_{AO}}{C_{AO}} \cdot \left(1 + \frac{m}{R_{AO}} \cdot y\right)^2 \quad \dots (3.14)$$

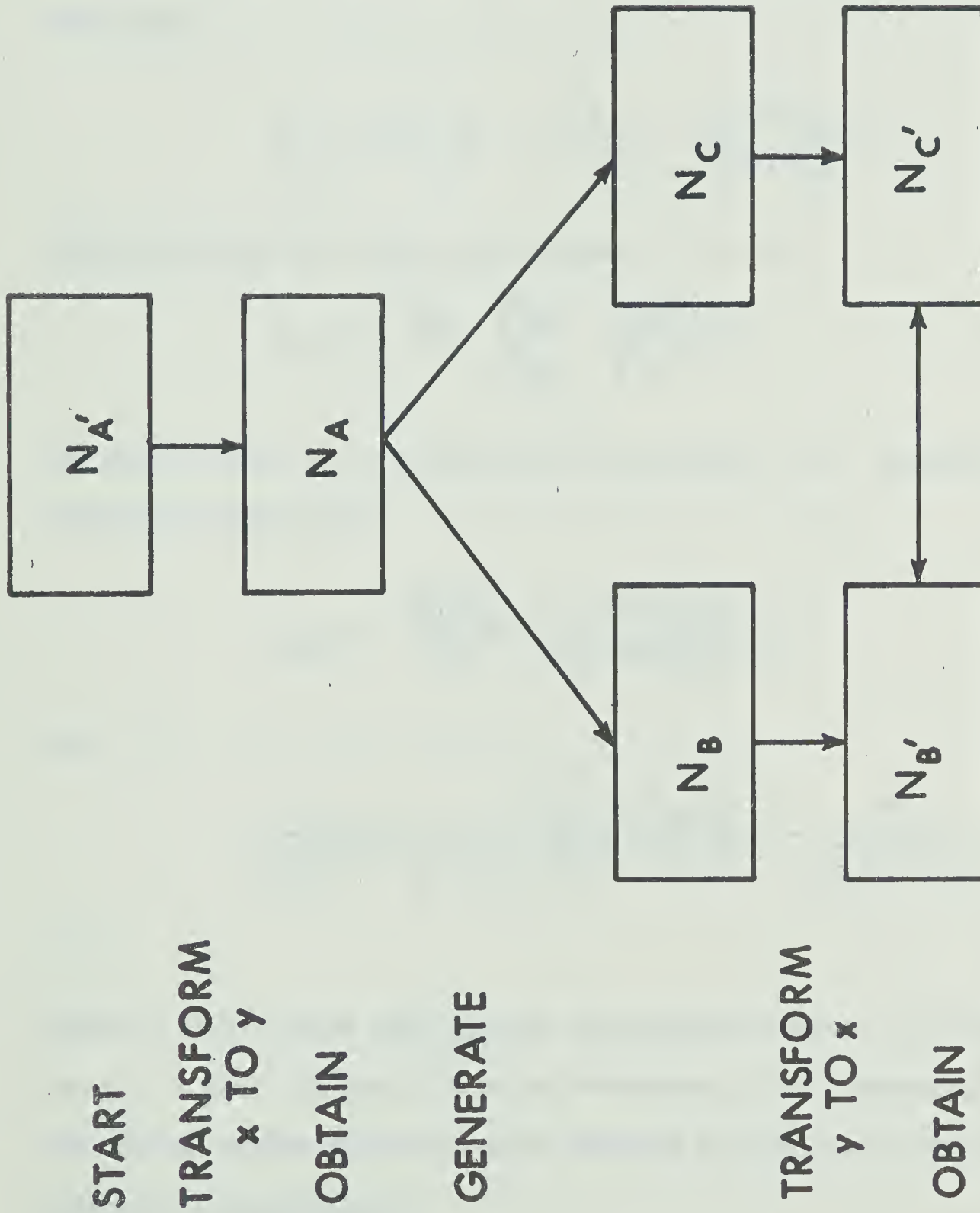


FIG 3.4 FLOW CHART OF EXAMPLE 3.1

or

$$y = \frac{R_{AO}}{m} \cdot \left[\left(\frac{Z_A(y)/R_{AO}}{Y_A(y)/C_{AO}} \right)^{\frac{1}{2}} - 1 \right] \quad \dots (3.15)$$

It is worth while to get some relationships between the equivalent structures N_A , and N_B . This can be obtained by using the following relations,

$$\int_0^x Z_{B'}(x) \cdot dx = y = \frac{R_{AO}}{m} \cdot \left[\left(\frac{Z_B(y)/R_{AO}}{Y_B(y)/C_{AO}} \right)^{\frac{1}{2}} - 1 \right] \quad \dots (3.16)$$

Differentiating both sides with respect to x we get,

$$Z_{B'}(x) = \frac{R_{AO}}{m} \cdot \sqrt{\frac{C_{AO}}{R_{AO}}} \cdot \left(\frac{Z_{B'}(x)}{Y_{B'}(x)} \right)', \quad \dots (3.17)$$

The prime indicates differentiation with respect to x . Equation (3.17) can be put in another form,

$$Z_{B'}(x) = \sqrt{\frac{R_{AO} C_{AO}}{2m}} \cdot \frac{[Z_{B'}(x)/Y_{B'}(x)]'}{[Z_{B'}(x)/Y_{B'}(x)]^{\frac{1}{2}}} \quad \dots (3.18)$$

or

$$\frac{1}{\sqrt{Z_{B'}(x) \cdot Y_{B'}(x)}} \cdot \frac{[Z_{B'}(x)/Y_{B'}(x)]'}{[Z_{B'}(x)/Y_{B'}(x)]} = \frac{2m}{\sqrt{R_{A'O} \cdot C_{A'O}}} = \text{constant} \quad \dots (3.19)$$

Equation (3.19) gives the required relationship between $Z_{B'}(x)$ and $Y_{B'}(x)$ for N_B , to have the same electrical behaviour as the exponential structure. This result checks with the result obtained by Hellstrom [16] by using a different transformation.

Two generated structures will be considered, one with uniform resistance $[Z_B(y)]$ and the other with uniform capacitance $[Y_C(y)]$.

For the first structure let,

$$Z_B(y) = R_B = R_{BO} = R_{B'O} \quad \dots (3.20)$$

Using equations (3.14) and (3.20) we get,

$$Y_B(y) = \frac{R_{BO} C_{AO}}{R_{AO}} \cdot (1 + \frac{m}{R_{AO}} \cdot y)^{-2} \quad \dots (3.21)$$

By using the inverse transform we have,

$$x = \int_0^y \frac{1}{R_{BO}} \cdot dy = \frac{y}{R_{BO}} \quad \dots (3.22)$$

Substituting equation (3.22) in equation (3.21) we get,

$$Y_{B'}(x) = \frac{R_{BO} C_{AO}}{R_{AO}} \cdot (1 + \frac{m \cdot R_{BO}}{R_{AO}} \cdot x)^{-2} \quad \dots (3.23)$$

Let $Y_{B'}(0) = C_{B'O}$, then equation (3.23) becomes,

$$\frac{Y_{B'}(x)}{C_{B'O}} = (1 + \sqrt{\frac{R_{B'O} C_{B'O}}{R_{A'O} C_{A'O}}} \cdot m \cdot x)^{-2} \quad \dots (3.24)$$

where

$$C_{B'O} = \frac{R_{B'O} C_{A'O}}{R_{A'O}} \quad \dots (3.25)$$

For the second structure let,

$$Y_C(y) = C_C = C_{c0} = C_{c'O} \quad \dots (3.26)$$

By using equation (3.14) and (3.26) we get,

$$Z_c(y) = \frac{C_{cO} \cdot R_{AO}}{C_{AO}} \left(1 + \frac{m}{R_{AO}} \cdot y\right)^2 \quad \dots (3.27)$$

Using the inverse transformation we have,

$$x = \int_0^y \frac{C_{AO}}{C_{cO} R_{AO}} \left(1 + \frac{m}{R_{AO}} \cdot y\right)^{-2} \cdot dy \quad \dots (3.28)$$

From equation (3.28) it can be easily shown that,

$$\left(1 + \frac{m}{R_{AO}} \cdot y\right) = \left(1 - \frac{C_{cO}}{C_{AO}} \cdot m \cdot x\right)^2 \quad \dots (3.29)$$

Substituting equation (3.29) in equation (3.27) we get,

$$Z_{c'}(x) = \frac{C_{c'O} R_{A'O}}{C_{A'O}} \cdot \left(1 - \frac{C_{c'O}}{C_{A'O}} \cdot m \cdot x\right)^{-2} \quad \dots (3.30)$$

Let $Z_{c'}(0) = R_{c'O}$, then equation (3.30) becomes,

$$\frac{Z_{c'}(x)}{R_{c'O}} = \left(1 - \sqrt{\frac{R_{c'O} C_{c'O}}{R_{A'O} C_{A'O}}} \cdot m \cdot x\right)^{-2} \quad \dots (3.31)$$

where

$$R_{c'O} = \frac{C_{c'O} R_{A'O}}{C_{A'O}} \quad \dots (3.32)$$

The results given by equation (3.24) and (3.31) are identical to the results of Hellstrom, referred to earlier.

The next step is the calculation of the lengths at which the three structures are equivalent. We know,

$$y_A = \int_0^{L_{A'}} R_{A'O} \exp(m \cdot x) \cdot dx \quad \dots (3.33)$$

$$y_B = \int_0^{L_{B'}} R_{B'O} \cdot dx \quad \dots (3.34)$$

$$y_C = \int_0^{L_{C'}} R_{C'O} \left(1 - \sqrt{\frac{R_{C'O} C_{C'O}}{R_{A'O} C_{A'O}}} \cdot m \cdot x\right)^{-2} \cdot dx \quad \dots (3.35)$$

For equivalence $y_A = y_B = y_C$. Therefore by equating equations (3.33) and (3.34) we get,

$$L_{B'} = \frac{1}{m} \cdot \sqrt{\frac{R_{A'O} C_{A'O}}{R_{B'O} C_{B'O}}} [\exp(m \cdot L_{A'}) - 1] \quad \dots (3.36)$$

Similarly, equating equations (3.33) and (3.35) we get,

$$L_{C'} = \frac{1}{m} \cdot \sqrt{\frac{R_{A'O} C_{A'O}}{R_{C'O} C_{C'O}}} [1 - \exp(-m \cdot L_{A'})] \quad \dots (3.37)$$

Thus equations (3.36) and (3.37) give the lengths (in terms of $L_{A'}$) for structures N_B , and N_C , respectively, at which they are equivalent to the exponential structures. These results check with Hellstrom's results.

Example 3.2

In this example we shall start with a given (Y/Z) ratio as a function of y . Three equivalent structures N_A , N_B , and N_C will be generated.

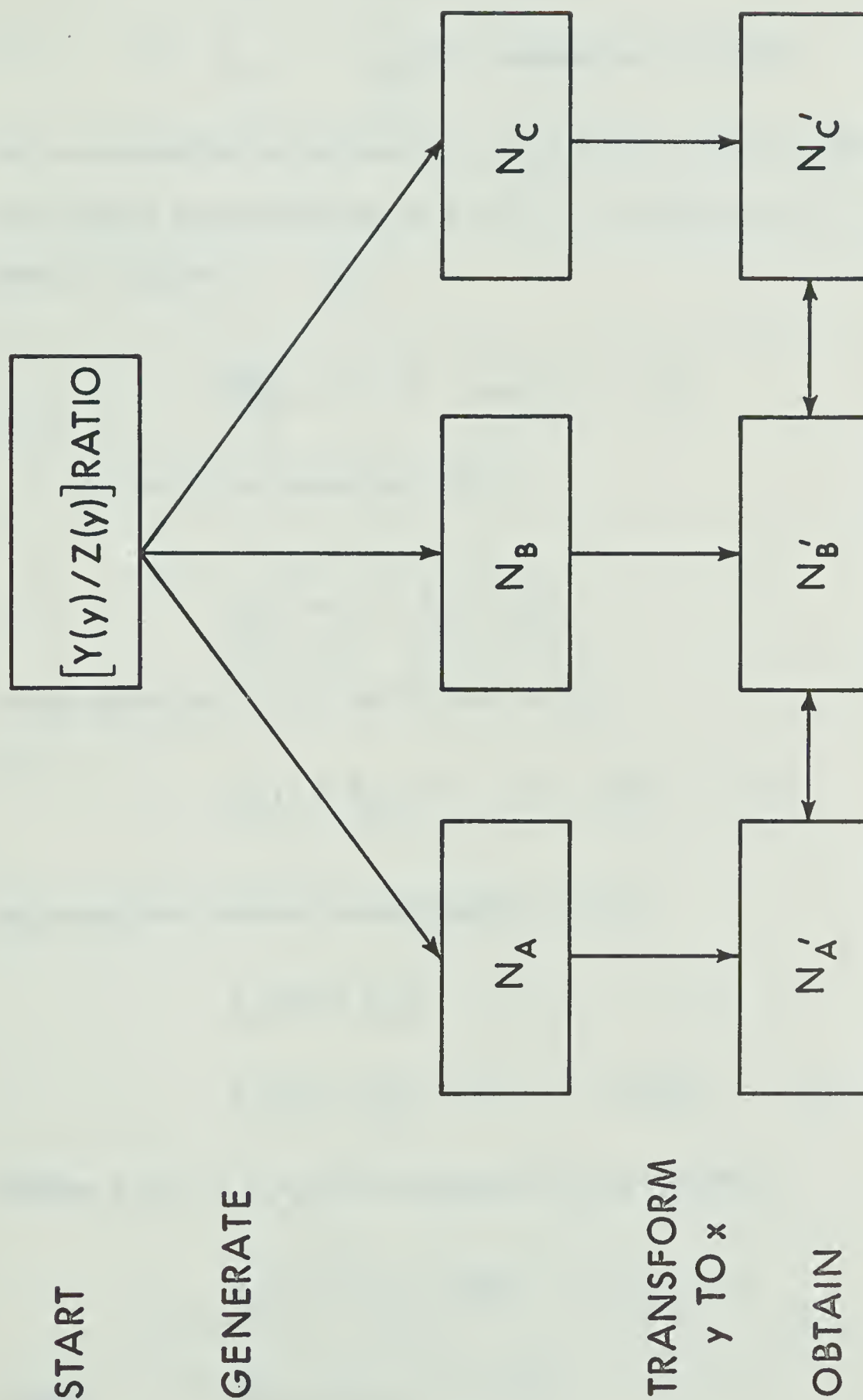


FIG 3.3 FLOW CHART OF EXAMPLE 3.2

$$\text{In } N_A, \quad Z_A(y) = \text{constant},$$

$$\text{In } N_B, \quad Y_B(y) = \text{constant},$$

$$\text{In } N_C, \quad Z_C(y) = \text{exponential function}.$$

The corresponding structures $N_{A'}$, $N_{B'}$, and $N_{C'}$, will be obtained by using the inverse transformation of y to x . The flow chart for this case is shown in Figure 3.5. Let,

$$\frac{Y(y)}{Z(y)} = \alpha^2 \cdot \beta^2 \cdot \exp(2 \cdot \beta \cdot y) \quad \dots (3.38)$$

For the first structure let,

$$Z_A(y) = R_A = R_{AO} = R_{A'O} \quad \dots (3.39)$$

Using equations (3.38) and (3.39) we get,

$$Y_A(y) = R_{AO} \cdot \alpha^2 \cdot \beta^2 \cdot \exp(2 \cdot \beta \cdot y) \quad \dots (3.40)$$

By using the inverse transformation we get,

$$Z_{A'}(x) = R_{A'O} \quad \dots (3.41)$$

$$Y_{A'}(x) = R_{A'O} \cdot \alpha^2 \cdot \beta^2 \cdot \exp(2 \cdot \beta \cdot R_{A'O} \cdot x) \quad \dots (3.42)$$

Assume $Y_{A'}(0) = C_{A'O}$, then equation (3.42) becomes,

$$Y_{A'}(x) = C_{A'O} \exp(2 \cdot \beta \cdot R_{A'O} \cdot x) \quad \dots (3.43)$$

where $C_{A'O} = R_{A'O} \cdot \alpha^2 \cdot \beta^2$

For the second structure let,

$$Y_B(y) = C_B = C_{BO} = C_{B'O} \quad \dots (3.44)$$

Then,

$$Z_B(y) = \frac{C_{BO}}{\alpha^2 \cdot \beta^2} \cdot \exp(-2 \cdot \beta \cdot y) \quad \dots (3.45)$$

By transforming y to x using the inverse transformation we get,

$$Z_{B'}(x) = R_{B'O} (1 + 2 \cdot \beta \cdot R_{B'O} \cdot x)^{-1} \quad \dots (3.46)$$

where

$$Z_{B'}(0) = R_{B'O} = \frac{C_{B'O}}{\alpha^2 \cdot \beta^2}$$

For the third structure let,

$$Z_c(y) = R_{cO} \exp(-\beta \cdot y) \quad \dots (3.47)$$

then

$$Y_c(y) = R_{cO} \cdot \alpha^2 \cdot \beta^2 \cdot \exp(\beta \cdot y) \quad \dots (3.48)$$

Again transforming y to x we get,

$$Z_{c'}(x) = R_{c'O} (1 + R_{c'O} \cdot \beta \cdot x)^{-1} \quad \dots (3.49)$$

$$Y_{c'}(x) = C_{c'O} (1 + R_{c'O} \cdot \beta \cdot x) \quad \dots (3.50)$$

where

$$R_{c'O} = R_{cO} \quad \text{and} \quad C_{c'O} = R_{c'O} \cdot \alpha^2 \cdot \beta^2$$

Now we will be able to obtain some relationships between the three lengths at which the three structures are equivalent. We have,

$$y_A = \int_0^{L_{A'}} R_{A'O} \cdot dx \quad \dots (3.51)$$

$$y_B = \int_0^{L_{B'}} \frac{R_{B'O}}{(1 + 2 \cdot \beta \cdot R_{B'O} \cdot x)} \cdot dx \quad \dots (3.52)$$

$$y_C = \int_0^{L_{C'}} \frac{R_{C'O}}{(1 + R_{C'O} \cdot \beta \cdot x)} \cdot dx \quad \dots (3.53)$$

For equivalence $y_A = y_B = y_C$. Then by equating equations (3.51) and (3.53) and equating equations (3.52) and (3.53) we get,

$$L_{A'} = \frac{1}{R_{A'O} \cdot \beta} \cdot \text{Log} (1 + R_{C'O} \cdot \beta \cdot L_{C'}) \quad \dots (3.54)$$

and

$$L_{B'} = \frac{1}{2R_{B'O} \cdot \beta} \cdot [(1 + R_{C'O} \cdot \beta \cdot L_{C'})^2 - 1] \quad \dots (3.55)$$

respectively.

While the distribution $[Y(y)/Z(y)]$ given in equation (3.38) was obtained by letting $v = 0$ in item 5, Table 2.1, it is interesting to note that the final results (especially the distributions $Z(x)$ and $Y(x)$) turn out to be similar to the distributions used by Hellstrom (example 3). It is encouraging to note that the results obtained by our procedure agree with those obtained by Hellstrom.

It is a matter of normalization; let $R_{A'O} = R_{B'O} = R_{C'O} = R_O$, and $C_{A'O} = C_{B'O} = C_{C'O} = C_O$. Then Figure 3.6 represents different distributions of $Z(x)$ and $Y(x)$ for the three structures, and equations (3.51), (3.52), (3.53). It is worth noting that for the same value of y the three structures

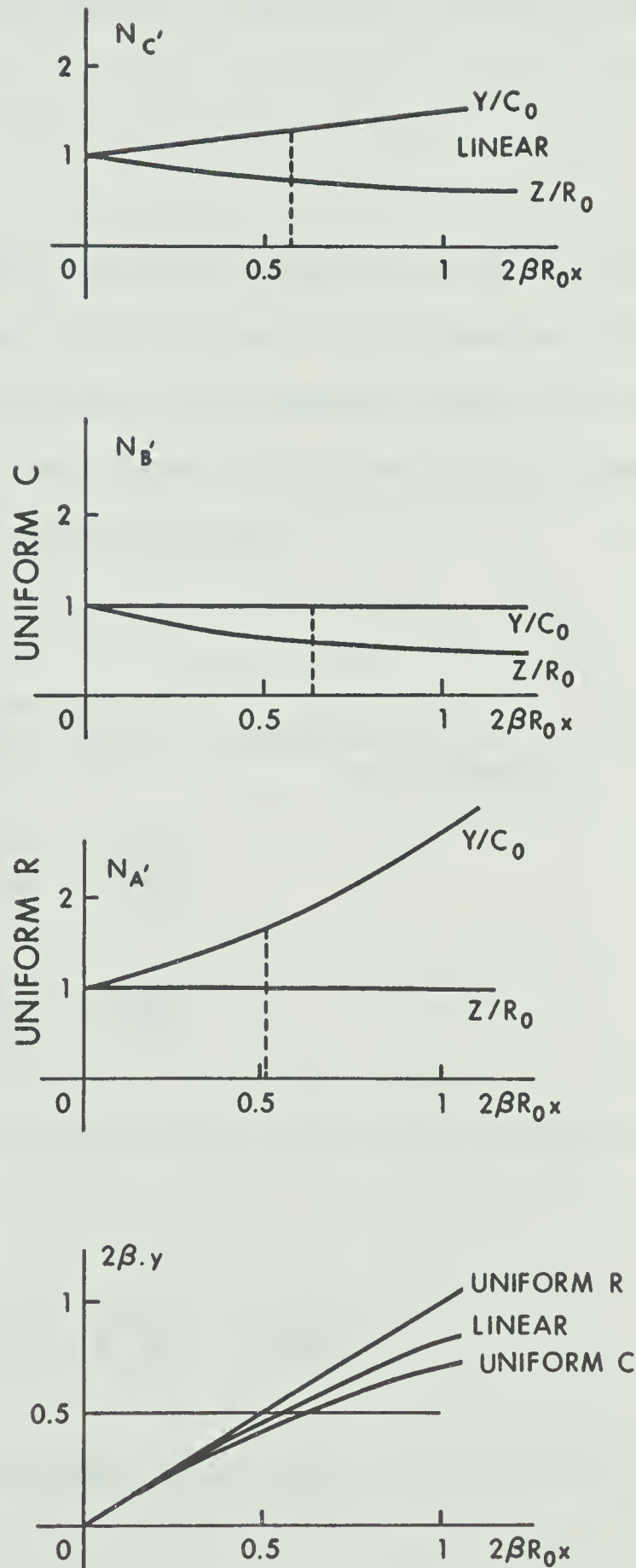


FIG 3.6 RELATIONSHIPS BETWEEN THE VARIABLES x AND y AND THE CORRESPONDING DISTRIBUTIONS FOR THE THREE STRUCTURES (EXAMPLE 3.2)

have the same (Y/Z) ratio, at the appropriate lengths; secondly the areas under the Z(x) and Y(x) curves are equal for the three cases since total resistance and capacitance for equivalent structures are equal, as it is proved earlier.

Example 3.3

This example can be considered as a special case of the previous example by assuming $[Y(y)/Z(y)]$ ratio to be constant. This represents a class of structures in which the impedance function and admittance function are proportional to each other. Hellstrom [17] has shown that they are equivalent to the uniform structure.

Let,

$$\left. \begin{aligned} \text{Let, } \frac{Y_A(y)}{Z_A(y)} &= \frac{k_1}{k_2} = k, \quad \text{where } k_1, k_2, k \\ &\quad \text{are constants.} \\ \frac{Y_B(y)}{Z_B(y)} &= \frac{k f(y)}{f(y)} \\ \text{Let, } \frac{Y_C(y)}{Z_C(y)} &= \frac{k g(y)}{g(y)} \end{aligned} \right\} \dots (3.56)$$

By following the procedure used in the previous example it can be easily shown that,

$$\frac{Y_{A'}(x)}{Z_{A'}(x)} = \frac{Y_{B'}(x)}{Z_{B'}(x)} = \frac{Y_{C'}(x)}{Z_{C'}(x)} = k \quad \dots (3.52)$$

The three lengths are related to each other by the following relations,

$$k_2 \cdot L_{A'} = \int_0^{L_{B'}} f(x) \cdot dx \quad \dots (3.58)$$

and

$$k_2 \cdot L_{A'} = \int_0^{L_{C'}} g(x) \cdot dx \quad \dots (3.59)$$

if the uniform line has a resistance per unit length = k_2

A new method of analyzing an R-C distributed structure has been presented in this thesis. The differential equation with variable coefficients which describes a distributed R-C structure under sinusoidal steady state conditions is solved by a transformation of the independent variable (space variable x), treating the frequency of the signals as a parameter, which can be assigned specific values. This method of solving the differential equation has certain advantages over the method of Gough and Gould [1]. The main advantage of this transformation lies in the fact that the solution of the transformed equation can be obtained very easily by comparing the transformed equation with equations whose solutions are known. Another advantage which results is that several distributed structures with the same electrical characteristics can be generated, some of which may be more suitable for fabrication. Such distributed structures are said to be equivalent.

This concept of equivalence has been explored in some detail by considering a number of examples. It is found that the conclusions arrived at are the same as those of Hellstrom [16] who used a different transformation.

A procedure for obtaining the frequency response of a distributed R-C structure has been presented. This computational procedure involving the use of a digital computer is illustrated for a few specific distributions $Z(x)$ and $Y(x)$. This procedure becomes very useful in view of the concept of equivalence mentioned earlier. If the frequency response of one structure is known, then the response of several equivalent structures is

also known.

In the case of a structure with uniform resistance and decreasing capacitance it is found that this structure possesses very sharp cut-off characteristic.

There is still much work which has to be done in this area. For instance, it will be worth while to investigate the electrical characteristics of structures with different distributions.

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APPENDIX 1

MATHEMATICAL MODEL OF A A DISTRIBUTED R-C STRUCTURE

Figure 1.1 gives the lumped approximation for an incremental length dx of the given distributed R-C structure, where $r(x)$ is the resistance per unit length and $c(x)$ is the capacitance per unit length. The circuit equations are,

$$v(x,t) - r(x) \cdot \Delta x \cdot i(x + \Delta x, t) - v(x + \Delta x, t) = 0 \quad \dots (A1.1)$$

$$i(x,t) - c(x) \cdot \Delta x \cdot \frac{\partial v(x,t)}{\partial t} - i(x + \Delta x, t) = 0 \quad \dots (A1.2)$$

Let Δx tend to zero. Then equation (A1.1) and equation (A1.2) reduce to,

$$\frac{\partial v}{\partial x} = -r(x) \cdot i \quad \dots (A1.3)$$

$$\frac{\partial i}{\partial x} = -c(x) \cdot \frac{\partial v}{\partial t} \quad \dots (A1.4)$$

Eliminating $i(x,t)$ between equation (A1.3) and equation (A1.4) we get,

$$\frac{\partial}{\partial x} \left[-\frac{1}{r(x)} \cdot \frac{\partial v}{\partial x} \right] = -c(x) \cdot \frac{\partial v}{\partial t} \quad \dots (A1.5)$$

or

$$\frac{\partial^2 v}{\partial x^2} - \frac{1}{r(x)} \cdot \frac{dr(x)}{dx} \cdot \frac{\partial v}{\partial x} = r(x) \cdot c(x) \cdot \frac{\partial v}{\partial t} \quad \dots (A1.6)$$

The solution of this equation gives the voltage along the line as a function of x . By choosing R and C as constants, equation (A1.6) becomes the classical one-dimensional linear heat flow or diffusion equation.

The general form of a second order differential equation with variable coefficients is,

$$\frac{d^2\psi}{dt^2} + P(t) \frac{d\psi}{dt} + Q(t) \cdot \psi = 0 \quad \dots (A2.1)$$

Let us transform the independent variable t to T , then,

$$\left. \begin{aligned} \frac{d\psi}{dt} &= \frac{d\psi}{dT} \cdot \frac{dT}{dt} \\ \frac{d^2\psi}{dt^2} &= \frac{d^2\psi}{dT^2} \left(\frac{dT}{dt}\right)^2 + \frac{d\psi}{dT} \cdot \left(\frac{d^2T}{dt^2}\right) \end{aligned} \right\} \quad \dots (A2.2)$$

By substituting these relationships in equation (A2.1) we get,

$$\frac{d^2\psi}{dT^2} \left(\frac{dT}{dt}\right)^2 + \left[\frac{d^2T}{dt^2} + P(t) \frac{dT}{dt}\right] \frac{d\psi}{dT} + Q(t) \cdot \psi = 0 \quad \dots (A2.3)$$

Let the coefficient of $\frac{d\psi}{dT} = 0$, then equation (A2.3) reduces to,

$$\frac{d^2\psi}{dT^2} \left(\frac{dT}{dt}\right)^2 + Q(T) \cdot \psi = 0 \quad \dots (A2.4)$$

Equating the coefficient of $\frac{d\psi}{dT}$ to zero gives,

$$\frac{d^2T}{dt^2} + P(t) \cdot \frac{dT}{dt} = 0 \quad \dots (A2.5)$$

Equation (A2.5) has a solution, namely,

$$T = \int_0^t e^{-\int P(\tau) d\tau} d\tau \quad \dots (A2.6)$$

Replacing t by x , T by y , τ by λ , and $p(\tau)$ by $-\frac{d}{d\lambda} \log Z(\lambda)$ in equation (A2.6) we get,

$$\begin{aligned} y &= \int_0^x e^{\int \frac{d}{d\lambda} \log Z(\lambda) d\lambda} d\lambda \\ &= \int_0^x Z(\lambda) \cdot d\lambda \quad \dots (A2.7) \end{aligned}$$

which is the same as equation (2.3).

APPENDIX 3

THE MATRIX PARAMETERS OF THE DISTRIBUTED R-C STRUCTURE

In this Appendix we shall give a summary of Dutta Roy's work [15]. We shall treat the R-C structure as a two-port network. By substituting the boundary conditions in equation (2.10) and equation (2.14) we get,

$$V(o) = k_1 p(o) + k_2 q(o) = V_1 \quad \dots (A3.1)$$

$$V(L) = k_1 p(L) + k_2 q(L) = V_2 \quad \dots (A3.2)$$

$$I(o) = \frac{-1}{Z(o)} \cdot [k_1 p'(o) + k_2 q'(o)] = I_1 \quad \dots (A3.3)$$

$$I(L) = -\frac{1}{Z(L)} [k_1 p'(L) + k_2 q'(L)] = -I_2 \quad \dots (A3.4)$$

where V_1 , V_2 , I_1 , and I_2 represent the amplitudes of steady state sinusoidal voltages and currents. The constants k_1 and k_2 may be evaluated from any two of the above equations. If these values are substituted in the other two equations, we get a set of equations relating V_1 , V_2 , I_1 , and I_2 . These equations can be used to obtain the matrix parameters of the distributed R-C structure. These expressions for these parameters are tabulated in Table A3.1. For convenience and brevity the following notations have been used by Dutta Roy,

$$\left. \begin{aligned} m_1 &= p'(L) \cdot q(o) - q'(L) \cdot p(o) \\ m_2 &= p'(o) \cdot q'(L) - q'(o) \cdot p'(L) \\ m_3 &= p'(o) \cdot q(o) - q'(o) \cdot p(o) \end{aligned} \right\} \quad \dots (A3.5)$$

$$\left. \begin{aligned} m_4 &= p'(L) \cdot q(1) - q'(L) \cdot p(L) \\ m_5 &= p'(0) \cdot q(L) - q'(0) \cdot p(L) \\ m_6 &= p(0) \cdot q(L) - q(0) \cdot p(L) \end{aligned} \right\} \dots \text{ (A3.5)}$$

MATRIX DESIGNATION	DEFINING EQUATIONS	PARAMETER VALUES
$[Z]$	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [Z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$	$\frac{1}{m_2} \begin{bmatrix} m_1 \cdot Z(o) & m_3 \cdot Z(L) \\ m_4 \cdot Z(o) & m_5 \cdot Z(L) \end{bmatrix}$
$[Y]$	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [Y] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$	$\frac{1}{m_6} \begin{bmatrix} -m_5/Z(o) & m_3/Z(o) \\ m_4/Z(L) & -m_1/Z(L) \end{bmatrix}$
$[a]$	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$	$\frac{1}{m_4} \begin{bmatrix} m_1 & -m_6 \cdot Z(L) \\ m_2/Z(o) & m_5 \cdot Z(L)/Z(o) \end{bmatrix}$
$[g]$	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = [g] \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$	$\frac{1}{m_1} \begin{bmatrix} m_2/Z(o) & -m_3 \cdot Z(L)/Z(o) \\ m_4 & -m_6 \cdot Z(L) \end{bmatrix}$
$[h]$	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = [h] \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$	$\frac{1}{m_5} \begin{bmatrix} -m_6/Z(o) & m_3 \\ -m_4 \cdot Z(o)/Z(L) & m_2 \cdot Z(L) \end{bmatrix}$

TABLE A3.1 MATRIX PARAMETERS

The numerical computations for determining the frequency response of the distributed R-C structure with $(a + bx)^Q$ as the $[Y(x)/Z(x)]$ ratio for different values of a , b and Q were carried out on the IBM 360 computer. The frequency response curves based on these computer results are shown in Figures 2.2 to 2.6. The detailed computer results are included in this Appendix.

The following nomenclature is used:

$$Q = Q$$

$$A = a$$

$$B = b$$

$$x_o = \text{initial value for } x$$

$$x_L = \text{final value for } x$$

$$W = \text{frequency}$$

$$\left. \begin{array}{l} Z_{11} \\ Z_{22} \\ Z_{12} \end{array} \right\} = \underline{Z} \text{ parameters}$$

$$V_2/V_1 = \left. \frac{V_2}{V_1} \right|_{I_2 = 0}$$

$$TF \cdot M = \left. \frac{V_2}{V_1} \right|_{I_2 = 0}$$

$$TF \cdot A = \left. \frac{V_2}{V_1} \right|_{I_2 = 0} \text{ in radians}$$

$$TF \cdot DB = 20 \log_{10} \left. \frac{V_2}{V_1} \right|_{I_2 = 0}$$

$$TF \cdot A = \left. \frac{V_2}{V_1} \right|_{I_2 = 0} \text{ in degrees}$$

Q = 0.0
A = 1.000
B = 2.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.D8	TF.A
2.0000	0.3252	-0.5428	0.3252	-0.5428	0.7731	5.4129	-2.2350	310.1340
4.0000	0.3042	-0.3272	-0.1386	0.0901	0.5151	4.8883	-5.7620	280.0767
6.0000	0.2774	-0.2666	-0.1130	-0.0622	0.3646	4.5409	-8.7625	260.1750
8.0000	0.2508	-0.2372	-0.0882	-0.1175	0.2739	4.2692	-11.2475	244.5045
10.0000	0.2273	-0.2174	-0.0668	-0.1342	0.2143	4.0360	-13.3785	231.7454
12.0000	0.2077	-0.2017	-0.0497	-0.1336	0.1724	3.8264	-15.2675	219.2351
14.0000	0.1916	-0.1884	-0.0365	-0.1248	0.1415	3.6332	-16.9840	208.1688
16.0000	0.1785	-0.1770	-0.0264	-0.1122	0.1179	3.4527	-18.5713	197.8260
18.0000	0.1677	-0.1672	-0.0188	-0.0984	0.0993	3.2825	-20.0577	188.0727
20.0000	0.1587	-0.1587	-0.0131	-0.0845	0.0845	3.1210	-21.4621	178.8184
22.0000	0.1510	-0.1513	-0.0089	-0.0714	0.0725	2.9670	-22.7979	169.9976
24.0000	0.1444	-0.1447	-0.0057	-0.0593	0.0626	2.8197	-24.0749	161.5560
26.0000	0.1386	-0.1390	-0.0034	-0.0486	0.0543	2.6782	-25.3007	153.4509
28.0000	0.1335	-0.1338	-0.0016	-0.0392	0.0474	2.5421	-26.4806	145.6497
30.0000	0.1290	-0.1292	-0.0004	-0.0310	0.0416	2.4106	-27.6197	138.1188
32.0000	0.1249	-0.1251	0.0005	-0.0240	0.0366	2.2836	-28.7218	130.8406
34.0000	0.1212	-0.1213	0.0011	-0.0180	0.0324	2.1604	-29.7911	123.7818
36.0000	0.1178	-0.1179	0.0015	-0.0130	0.0287	2.0408	-30.8281	116.9272
38.0000	0.1146	-0.1147	0.0017	-0.0089	0.0256	1.9245	-31.8391	110.2642
40.0000	0.1118	-0.1118	0.0019	-0.0054	0.0228	1.8111	-32.8228	103.7679
42.0000	0.1091	-0.1091	0.0019	-0.0026	0.0205	1.7007	-33.7827	97.4402
44.0000	0.1066	-0.1066	0.0019	-0.0004	0.0184	1.5929	-34.7183	91.2644
46.0000	0.1042	-0.1043	0.0019	0.0014	0.0165	1.4875	-35.6347	85.2250
48.0000	0.1021	-0.1020	0.0018	0.0028	0.0149	1.3842	-36.5293	79.3077
50.0000	0.1000	-0.1000	0.0017	0.0038	0.0135	1.2834	-37.4085	73.5343

Q = 0.0
A = 2.000
B = 1.000
X0= 0.0
XL= 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.D8	TF.A				
2.0000	0.3252	-0.5428	0.3252	-0.5428	-0.1588	-0.4627	0.4984	-0.5911	0.7731	5.4129	-2.2348	310.1348
4.0000	0.3043	-0.3274	0.3042	-0.3273	-0.1386	-0.1838	0.0901	-0.5070	0.5150	4.8883	-5.7642	280.0306
6.0000	0.2776	-0.2660	0.2774	-0.2665	-0.1129	-0.0831	-0.0625	-0.3594	0.3648	4.5401	-8.7595	260.1301
8.0000	0.2511	-0.2347	0.2506	-0.2371	-0.0879	-0.0343	-0.1185	-0.2476	0.2745	4.2658	-11.2295	244.4146
10.0000	0.2248	-0.2158	0.2269	-0.2172	-0.0665	-0.0087	-0.1346	-0.1680	0.2153	4.0369	-13.3387	231.2989
12.0000	0.2138	-0.2032	0.2084	-0.2012	-0.0501	0.0041	-0.1327	-0.1071	0.1705	3.8204	-15.3639	218.8907
14.0000	0.1700	-0.1735	0.1882	-0.1868	-0.0346	0.0106	-0.1310	-0.0713	0.1492	3.6399	-16.5270	208.5519
16.0000	0.1734	-0.2127	0.1788	-0.1810	-0.0268	0.0158	-0.1064	-0.0393	0.1134	3.4952	-18.9092	200.2603
18.0000	0.1905	-0.1762	0.1912	-0.1923	-0.0203	0.0153	-0.0975	-0.0096	0.0980	3.2401	-20.1761	185.6422
20.0000	0.0789	-0.3284	0.1591	-0.2164	-0.0098	0.0217	-0.0693	-0.0131	0.0705	3.3291	-23.0363	190.7412
22.0000	0.2426	-0.0178	0.1811	-0.1254	-0.0127	0.0086	-0.0547	0.0312	0.0630	2.6224	-24.0151	150.2551
24.0000	-0.0938	-0.2188	0.1750	-0.2250	0.0017	0.0129	-0.0528	-0.0148	0.0548	3.4152	-25.2178	195.6746
26.0000	-0.0625	-0.5000	0.3500	-0.3625	0.0086	0.0211	-0.0437	0.0117	0.0452	2.8791	-26.8967	154.9587
28.0000	-0.1016	0.1484	0.1563	-0.1563	-0.0007	0.0012	0.0079	-0.0005	0.0079	6.2236	-42.0310	356.5833
30.0000	-0.0595	0.0696	0.1350	-0.1014	-0.0006	0.0015	0.0164	-0.0055	0.0173	5.9614	-35.2422	341.5649
32.0000	-0.1108	0.2005	0.1769	-0.0566	-0.0019	-0.0012	-0.0007	0.0098	0.0098	1.5469	-40.1613	94.3595
34.0000	-0.0779	0.0644	0.1183	-0.1163	0.0001	0.0010	0.0051	-0.0081	0.0096	5.2791	-40.3663	302.4709
36.0000	-0.0781	0.0781	0.1887	-0.1310	0.0002	0.0008	0.0038	-0.0067	0.0078	5.2315	-42.2115	299.7446
38.0000	-0.0987	0.1024	0.1082	-0.1409	0.0002	0.0005	0.0018	-0.0034	0.0038	5.2012	-48.3687	298.0090
40.0000	-0.0964	0.0834	0.1170	-0.1181	0.0003	0.0003	-0.0007	-0.0033	0.0033	4.5150	-49.5781	258.6897
42.0000	-0.0672	0.1607	0.0787	-0.0844	-0.0003	-0.0001	-0.0000	0.0020	0.0020	1.5708	-54.1854	90.0001
44.0000	-0.0680	0.1471	0.1195	-0.0404	-0.0001	0.0005	0.0031	-0.0006	0.0031	6.0950	-50.1550	349.2153
46.0000	-0.1029	0.1299	0.0929	-0.0951	-0.0003	-0.0000	0.0011	0.0014	0.0017	0.9040	-55.1666	51.7945
48.0000	-0.1015	0.0769	0.1015	-0.0769	-0.0001	-0.0000	0.0007	0.0007	0.0009	0.7854	-60.7175	45.0000
50.0000	-0.1581	-0.0588	0.1287	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-INFINITY	0.0

Q = 0.0
A = 2.000
B = 2.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A				
2.0000	0.3252	-0.5428	0.3252	-0.5428	-0.1588	-0.4627	0.4983	-0.5911	0.7731	5.4129	-2.2350	310.1340
4.0000	0.3042	-0.3272	0.3042	-0.3272	-0.1386	-0.1838	0.0901	-0.5072	0.5151	4.8883	-5.7620	280.0769
6.0000	0.2774	-0.2666	0.2774	-0.2666	-0.1130	-0.0831	-0.0622	-0.3593	0.3647	4.5409	-8.7623	260.1743
8.0000	0.2507	-0.2372	0.2508	-0.2372	-0.0882	-0.0342	-0.1175	-0.2475	0.2739	4.2692	-11.2471	244.6060
10.0000	0.2273	-0.2174	0.2273	-0.2174	-0.0668	-0.0088	-0.1342	-0.1671	0.2143	4.0360	-13.3785	231.2430
12.0000	0.2076	-0.2017	0.2077	-0.2017	-0.0497	0.0043	-0.1336	-0.1091	0.1724	3.8264	-15.2675	219.2375
14.0000	0.1916	-0.1885	0.1916	-0.1884	-0.0365	0.0107	-0.1247	-0.0668	0.1415	3.6333	-16.9845	208.1710
16.0000	0.1785	-0.1773	0.1785	-0.1771	-0.0264	0.0134	-0.1122	-0.0361	0.1178	3.4530	-18.5742	197.8425
18.0000	0.1680	-0.1674	0.1677	-0.1672	-0.0188	0.0141	-0.0983	-0.0139	0.0993	3.2824	-20.0638	188.0662
20.0000	0.1587	-0.1586	0.1587	-0.1587	-0.0131	0.0137	-0.0845	0.0018	0.0845	3.1207	-21.4615	178.8051
22.0000	0.1511	-0.1512	0.1510	-0.1513	-0.0089	0.0127	-0.0714	0.0126	0.0725	2.9667	-22.7985	169.9777
24.0000	0.1443	-0.1444	0.1444	-0.1447	-0.0057	0.0114	-0.0594	0.0198	0.0626	2.8192	-24.0663	161.5309
26.0000	0.1386	-0.1387	0.1386	-0.1389	-0.0034	0.0101	-0.0486	0.0243	0.0543	2.6778	-25.2968	153.4265
28.0000	0.1333	-0.1337	0.1335	-0.1338	-0.0016	0.0088	-0.0392	0.0268	0.0475	2.5420	-26.4749	145.6477
30.0000	0.1291	-0.1297	0.1292	-0.1292	-0.0004	0.0076	-0.0310	0.0277	0.0415	2.4114	-27.6289	138.1623
32.0000	0.1249	-0.1256	0.1250	-0.1249	0.0005	0.0065	-0.0240	0.0277	0.0366	2.2849	-28.7280	130.9173
34.0000	0.1213	-0.1206	0.1212	-0.1211	0.0011	0.0054	-0.0180	0.0270	0.0324	2.1597	-29.7828	123.7438
36.0000	0.1185	-0.1158	0.1179	-0.1176	0.0015	0.0045	-0.0129	0.0258	0.0288	2.0348	-30.8065	116.5871
38.0000	0.1137	-0.1107	0.1146	-0.1146	0.0017	0.0037	-0.0088	0.0243	0.0259	1.9176	-31.7475	109.8731
40.0000	0.1092	-0.1090	0.1117	-0.1117	0.0018	0.0030	-0.0055	0.0225	0.0231	1.8096	-32.7233	103.6797
42.0000	0.1062	-0.1077	0.1092	-0.1095	0.0019	0.0025	-0.0028	0.0205	0.0207	1.7047	-33.6924	97.6694
44.0000	0.1056	-0.1059	0.1063	-0.1062	0.0019	0.0020	-0.0004	0.0184	0.0184	1.5942	-34.6848	91.3412
46.0000	0.1040	-0.1056	0.1037	-0.1038	0.0019	0.0016	0.0013	0.0164	0.0165	1.4935	-35.6765	85.5705
48.0000	0.0982	-0.1026	0.1015	-0.1021	0.0018	0.0012	0.0026	0.0148	0.0150	1.3954	-36.4569	79.9490
50.0000	0.0986	-0.1024	0.0996	-0.0996	0.0017	0.0009	0.0037	0.0129	0.0134	1.2939	-37.4527	74.1344

Q = 1.000
A = 1.000
B = 1.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.D8	TF.A				
2.0000	0.3734	-0.4030	0.2676	-0.3836	-0.1472	-0.0006	0.0078	-0.0271	0.0365	2.4068	-28.7571	137.8978
4.0000	0.3305	-0.2812	0.2366	-0.2492	-0.1125	-0.0007	0.0062	-0.0182	0.0302	2.2195	-30.3978	127.1663
6.0000	0.2862	-0.2449	0.2047	-0.2075	-0.0776	-0.0014	0.0048	-0.0114	0.0252	2.0405	-31.9689	116.9129
8.0000	0.2504	-0.2215	0.1789	-0.1828	-0.0508	-0.0032	0.0036	-0.0062	0.0212	1.8688	-33.4742	107.0724
10.0000	0.2234	-0.2027	0.1594	-0.1646	-0.0321	-0.0119	0.0027	-0.0024	0.0178	1.7035	-34.9207	97.6028
12.0000	0.2032	-0.1872	0.1448	-0.1503	-0.0197	-0.0140	0.0019	0.0004	0.0153	1.5440	-36.3170	88.4660
14.0000	0.1876	-0.1744	0.1337	-0.1388	-0.0115	-0.0133	0.0013	0.0024	0.0131	1.3897	-37.6714	79.6249
16.0000	0.1753	-0.1638	0.1248	-0.1295	-0.0062	-0.0116	0.0009	0.0036	0.0112	1.2403	-38.9796	71.0656
18.0000	0.1653	-0.1549	0.1176	-0.1217	-0.0028	-0.0097	0.0005	0.0044	0.0097	1.0953	-40.2473	62.7586
20.0000	0.1568	-0.1474	0.1115	-0.1152	-0.0006	-0.0078	0.0003	0.0049	0.0084	0.9543	-41.4854	54.6791
22.0000	0.1496	-0.1409	0.1064	-0.1096	0.0007	-0.0062	0.0001	0.0050	0.0073	0.8160	-42.6935	46.7548
24.0000	0.1433	-0.1352	0.1019	-0.1048	0.0014	-0.0048	-0.0001	0.0050	0.0064	0.6821	-43.8772	39.0793
26.0000	0.1378	-0.1302	0.0979	-0.1006	0.0018	-0.0036	0.0003	0.0049	0.0056	0.5504	-45.0148	31.5365
28.0000	0.1329	-0.1257	0.0944	-0.0968	0.0019	-0.0027	0.0001	0.0050	0.0049	0.4220	-46.1350	24.1770
30.0000	0.1284	-0.1217	0.0912	-0.0934	0.0019	-0.0019	0.0003	0.0042	0.0044	0.2969	-47.2287	17.0088
32.0000	0.1244	-0.1181	0.0883	-0.0904	0.0018	-0.0013	0.0002	0.0038	0.0038	0.1750	-48.3157	10.0247
34.0000	0.1207	-0.1148	0.0857	-0.0877	0.0017	-0.0009	0.0002	0.0029	0.0029			
36.0000	0.1174	-0.1117	0.0832	-0.0851	0.0015	-0.0005	0.0002	0.0020	0.0020			
38.0000	0.1142	-0.1089	0.0810	-0.0828	0.0013	-0.0003	0.0001	0.0013	0.0013			
40.0000	0.1115	-0.1063	0.0790	-0.0807	0.0011	-0.0001	0.0001	0.0010	0.0010			
42.0000	0.1091	-0.1042	0.0771	-0.0787	0.0010	-0.0001	0.0001	0.0008	0.0008			
44.0000	0.1064	-0.1016	0.0753	-0.0769	0.0008	-0.0002	0.0002	0.0007	0.0007			
46.0000	0.1043	-0.0995	0.0737	-0.0751	0.0007	-0.0002	0.0002	0.0005	0.0005			
48.0000	0.1021	-0.0974	0.0721	-0.0735	0.0005	-0.0003	0.0003	0.0004	0.0004			
50.0000	0.1001	-0.0955	0.0707	-0.0720	0.0004	-0.0003	0.0003	0.0003	0.0003			

Q = 1.000
A = 1.000
B = 2.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A
2.0000	0.3932	-0.3406	0.2385	-0.3062	-0.1342	-0.0888	-0.0520	0.0017
4.0000	0.3319	-0.2610	0.2004	-0.2092	-0.0888	-0.0520	0.0017	0.0125
6.0000	0.2796	-0.2304	0.1679	-0.1742	-0.0888	-0.0520	0.0017	0.0137
8.0000	0.2427	-0.2073	0.1450	-0.1513	-0.0888	-0.0520	0.0017	0.0137
10.0000	0.2169	-0.1890	0.1291	-0.1354	-0.0888	-0.0520	0.0017	0.0137
12.0000	0.1982	-0.1744	0.1176	-0.1231	-0.0888	-0.0520	0.0017	0.0137
14.0000	0.1838	-0.1628	0.1088	-0.1135	-0.0888	-0.0520	0.0017	0.0137
16.0000	0.1723	-0.1534	0.1017	-0.1058	-0.0888	-0.0520	0.0017	0.0137
18.0000	0.1628	-0.1455	0.0960	-0.0995	-0.0888	-0.0520	0.0017	0.0137
20.0000	0.1547	-0.1388	0.0911	-0.0942	-0.0888	-0.0520	0.0017	0.0137
22.0000	0.1478	-0.1330	0.0869	-0.0897	-0.0888	-0.0520	0.0017	0.0137
24.0000	0.1417	-0.1279	0.0832	-0.0857	-0.0888	-0.0520	0.0017	0.0137
26.0000	0.1363	-0.1234	0.0799	-0.0823	-0.0888	-0.0520	0.0017	0.0137
28.0000	0.1314	-0.1193	0.0770	-0.0792	-0.0888	-0.0520	0.0017	0.0137
30.0000	0.1271	-0.1157	0.0744	-0.0765	-0.0888	-0.0520	0.0017	0.0137
32.0000	0.1231	-0.1123	0.0721	-0.0740	-0.0888	-0.0520	0.0017	0.0137
34.0000	0.1195	-0.1093	0.0699	-0.0717	-0.0888	-0.0520	0.0017	0.0137
36.0000	0.1162	-0.1065	0.0680	-0.0696	-0.0888	-0.0520	0.0017	0.0137
38.0000	0.1132	-0.1039	0.0662	-0.0677	-0.0888	-0.0520	0.0017	0.0137
40.0000	0.1104	-0.1015	0.0645	-0.0660	-0.0888	-0.0520	0.0017	0.0137
42.0000	0.1078	-0.0992	0.0629	-0.0644	-0.0888	-0.0520	0.0017	0.0137
44.0000	0.1054	-0.0971	0.0615	-0.0629	-0.0888	-0.0520	0.0017	0.0137
46.0000	0.1031	-0.0952	0.0601	-0.0614	-0.0888	-0.0520	0.0017	0.0137
48.0000	0.1009	-0.0933	0.0589	-0.0601	-0.0888	-0.0520	0.0017	0.0137
50.0000	0.0989	-0.0916	0.0577	-0.0589	-0.0888	-0.0520	0.0017	0.0137

Q = 2.000
A = 1.000
B = 1.000
X0 = 0.0
X1 = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.D8	TF.A
2.0000	0.4097	0.2149	-0.1256	-0.0136	0.3751	4.6760	-8.5169	267.9160
4.0000	0.3362	0.1753	-0.0754	-0.1106	0.1853	4.0724	-14.6446	233.3319
6.0000	0.2798	0.1450	-0.0397	-0.1000	0.1125	3.6186	-18.9760	207.3317
8.0000	0.2424	0.1250	-0.0194	-0.0744	0.0747	3.2286	-22.5325	184.9837
10.0000	0.2170	0.1114	-0.0084	-0.0504	0.0522	2.8806	-25.6468	165.0489
12.0000	0.1986	0.1016	-0.0027	-0.0316	0.0378	2.5643	-28.4590	146.9214
14.0000	0.1843	0.0941	0.0001	-0.0181	0.0280	2.2725	-31.0439	130.2058
16.0000	0.1728	0.0881	0.0014	-0.0089	0.0213	2.0008	-33.4485	114.6350
18.0000	0.1633	0.0831	0.0019	-0.0028	0.0164	1.7453	-35.7054	99.9986
20.0000	0.1552	0.0789	0.0019	0.0009	0.0128	1.5036	-37.8378	86.1497
22.0000	0.1482	0.0752	0.0017	0.0030	0.0102	1.2736	-39.8644	72.9706
24.0000	0.1420	0.0720	0.0015	0.0040	0.0081	1.0538	-41.7987	60.3756
26.0000	0.1366	0.0692	0.0012	0.0044	0.0066	0.8428	-43.6535	48.2877
28.0000	0.1318	0.0667	0.0009	0.0043	0.0053	0.6397	-45.4384	36.6518
30.0000	0.1274	0.0645	0.0007	0.0040	0.0044	0.4437	-47.1591	25.4223
32.0000	0.1234	0.0624	0.0005	0.0035	0.0036	0.2543	-48.8224	14.5731
34.0000	0.1198	0.0606	0.0004	0.0030	0.0030	0.0708	-50.4369	4.0572
36.0000	0.1165	0.0589	0.0003	0.0025	0.0025	6.1749	-52.0018	353.7971
38.0000	0.1135	0.0573	0.0002	0.0020	0.0021	6.0016	-53.5236	343.8669
40.0000	0.1106	0.0558	0.0001	0.0016	0.0018	5.8316	-55.0010	334.1282
42.0000	0.1080	0.0545	0.0000	0.0012	0.0015	5.6684	-56.4560	324.7771
44.0000	0.1055	0.0532	0.0000	0.0009	0.0013	5.5079	-57.8740	315.5776
46.0000	0.1033	0.0521	-0.0000	0.0006	0.0011	5.3481	-59.2453	306.4207
48.0000	0.1011	0.0510	-0.0000	0.0004	0.0009	5.1930	-60.5960	297.5374
50.0000	0.0991	0.0500	-0.0000	0.0003	0.0008	5.0423	-61.9239	288.9038

Q = 2.000
A = 1.000
B = 2.000
XC = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A
2.0000	0.4122	-0.2672	0.1605	-0.1798	-0.0826	-0.0380	-0.0990	-0.1564
4.0000	0.3120	-0.2253	0.1176	-0.1277	-0.0273	0.0116	-0.0751	-0.0170
6.0000	0.2596	-0.1956	0.0957	-0.1031	-0.0066	0.0113	-0.0370	0.0156
8.0000	0.2282	-0.1756	0.0829	-0.0883	0.0001	0.0067	-0.0139	0.0186
10.0000	0.2065	-0.1614	0.0742	-0.0785	0.0018	0.0033	-0.0025	0.0142
12.0000	0.1902	-0.1505	0.0678	-0.0713	0.0018	0.0014	0.0023	0.0090
14.0000	0.1773	-0.1418	0.0628	-0.0658	0.0014	0.0034	0.0037	0.0050
16.0000	0.1667	-0.1346	0.0588	-0.0614	0.0009	-0.0001	0.0036	0.0023
18.0000	0.1578	-0.1284	0.0554	-0.0577	0.0006	-0.0003	0.0030	0.0007
20.0000	0.1503	-0.1231	0.0526	-0.0546	0.0003	-0.0003	0.0022	-0.0002
22.0000	0.1438	-0.1185	0.0502	-0.0520	0.0001	-0.0003	0.0015	-0.0006
24.0000	0.1380	-0.1144	0.0480	-0.0497	0.0000	-0.0002	0.0009	-0.0008
26.0000	0.1329	-0.1107	0.0462	-0.0477	-0.0000	-0.0002	0.0005	-0.0007
28.0000	0.1284	-0.1074	0.0445	-0.0459	-0.0000	-0.0001	0.0002	-0.0006
30.0000	0.1242	-0.1044	0.0430	-0.0443	-0.0001	-0.0001	0.0000	-0.0005
32.0000	0.1205	-0.1017	0.0416	-0.0429	-0.0000	-0.0000	-0.0001	-0.0004
34.0000	0.1171	-0.0991	0.0404	-0.0415	-0.0000	-0.0000	-0.0001	-0.0003
36.0000	0.1139	-0.0968	0.0392	-0.0403	-0.0000	-0.0000	-0.0002	-0.0002
38.0000	0.1111	-0.0946	0.0382	-0.0392	-0.0000	0.0000	-0.0002	-0.0001
40.0000	0.1084	-0.0926	0.0372	-0.0382	-0.0000	0.0000	-0.0001	-0.0001
42.0000	0.1059	-0.0907	0.0363	-0.0373	-0.0000	0.0000	-0.0001	-0.0000
44.0000	0.1035	-0.0890	0.0355	-0.0364	-0.0000	0.0000	-0.0001	-0.0000
46.0000	0.1014	-0.0873	0.0347	-0.0356	-0.0000	0.0000	-0.0001	0.0000
48.0000	0.0993	-0.0857	0.0340	-0.0348	-0.0000	0.0000	-0.0001	0.0000
50.0000	0.0974	-0.0843	0.0333	-0.0341	-0.0000	0.0000	-0.0000	0.0000

Q = 2.000
A = 2.000
B = 1.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A				
1.0000	0.3346	-0.2787	0.2253	-0.2397	-0.1082	-0.0755	-0.0800	-0.2923	0.3030	4.4454	-10.3698	254.7027
3.0000	0.2033	-0.1865	0.1363	-0.1431	-0.0167	0.0139	-0.0787	-0.0038	0.0788	3.1897	-22.0712	182.7555
5.0000	0.1573	-0.1469	0.1051	-0.1094	0.0001	0.0069	-0.0216	0.0239	0.0322	2.3058	-29.8542	132.1135
7.0000	0.1329	-0.1258	0.0890	-0.0919	0.0019	0.0021	-0.0002	0.0155	0.0155	1.5867	-36.1761	90.9117
9.0000	0.1170	-0.1092	0.0786	-0.0806	0.0013	0.0003	0.0048	0.0068	0.0083	0.9574	-41.6001	54.8541
11.0000	0.1046	-0.1015	0.0710	-0.0728	0.0006	-0.0002	0.0043	0.0019	0.0047	0.4063	-46.5165	23.2779
13.0000	0.0990	-0.0933	0.0643	-0.0666	0.0002	-0.0003	0.0028	-0.0003	0.0028	6.1727	-50.9444	353.6677
15.0000	0.0882	-0.1006	0.0600	-0.0619	0.0000	-0.0002	0.0015	-0.0009	0.0017	5.7490	-55.3336	329.3909
17.0000	0.0768	-0.1115	0.0547	-0.0588	-0.0000	-0.0001	0.0007	-0.0008	0.0011	5.3798	-59.3706	308.2422
19.0000	0.0699	-0.1145	0.0491	-0.0630	-0.0001	-0.0001	0.0002	-0.0007	0.0007	4.9209	-62.6479	281.9441
21.0000	0.0765	-0.0895	0.0573	-0.0641	-0.0000	-0.0000	-0.0001	-0.0005	0.0005	4.5491	-66.7470	260.6431
23.0000	0.1316	0.1464	0.0439	-0.0240	-0.0000	-0.0000	-0.0002	-0.0001	0.0002	3.6776	-74.5393	210.7133
25.0000	-0.0115	-0.1954	0.0050	-0.0603	-0.0000	0.0000	-0.0000	-0.0001	0.0001	4.4774	-79.0623	256.5344
27.0000	0.0515	0.0538	0.0623	-0.0225	0.0000	0.0000	0.0003	0.0001	0.0003	0.2529	-70.4799	14.4898
29.0000	-0.0213	0.0590	0.0485	-0.0375	-0.0000	0.0000	0.0000	-0.0000	0.0000	5.9138	-101.1881	338.8384
31.0000	-0.2292	0.0625	0.0417	-0.0417	0.0000	-0.0000	-0.0000	0.0001	0.0001	2.1588	-83.3866	123.6901
33.0000	-0.0196	0.0638	0.0454	-0.0399	0.0000	0.0000	0.0000	-0.0002	0.0002	5.0053	-75.3999	286.7800
35.0000	0.0875	0.0500	0.0125	-0.0500	-0.0000	0.0000	0.0001	0.0002	0.0002	1.1760	-72.2472	67.3801
37.0000	-0.0308	0.0795	0.0205	0.0026	-0.0000	0.0000	0.0002	0.0002	0.0003	0.8622	-71.5414	49.3987
39.0000	0.0375	0.2375	0.0	0.0	-0.0001	-0.0000	-0.0001	0.0002	0.0002	1.9461	-72.6144	111.5015
41.0000	-0.0240	0.0699	0.0132	-0.0384	-0.0000	-0.0000	0.0000	0.0001	0.0001	1.5708	-84.2883	89.9299
43.0000	0.0312	0.1875	0.0	-0.0312	-0.0001	-0.0000	-0.0002	0.0006	0.0007	1.9513	-63.6438	111.8015
45.0000	-0.0567	0.0525	0.0240	-0.0370	0.0000	-0.0000	-0.0001	-0.0000	0.0001	3.4541	-77.0917	197.9081
47.0000	-0.0586	0.0509	0.0273	-0.0457	0.0000	0.0000	0.0000	-0.0001	0.0001	5.0191	-77.5956	287.5710
49.0000	-0.0337	0.1461	0.0344	0.0176	-0.0000	-0.0001	-0.0003	0.0002	0.0004	2.6224	-68.0975	150.2551
51.0000	-0.0656	0.0500	0.0031	0.0	0.0000	-0.0000	-0.0006	-0.0002	0.0006	3.5477	-64.2926	203.2677

Q = 2.000
A = 2.000
B = 2.000
X0 = 0.0
XL = 1.000

N	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.D8	TF.A
2.0000	0.2424	-0.2040	0.1250	-0.1337	-0.0194	0.0136	-0.0744	-0.0065
4.0000	0.1728	-0.1516	0.0881	-0.0925	0.0014	0.0047	-0.0089	0.0193
6.0000	0.1420	-0.1269	0.0720	-0.0749	0.0015	0.0005	0.0040	0.0071
8.0000	0.1234	-0.1116	0.0624	-0.0645	0.0005	-0.0003	0.0035	0.0009
10.0000	0.1106	-0.1010	0.0558	-0.0575	0.0001	-0.0002	0.0016	-0.0008
12.0000	0.1011	-0.0929	0.0510	-0.0524	-0.0000	-0.0001	0.0004	-0.0008
14.0000	0.0937	-0.0866	0.0472	-0.0484	-0.0000	-0.0000	-0.0001	-0.0005
16.0000	0.0878	-0.0814	0.0442	-0.0452	-0.0000	-0.0000	-0.0002	-0.0002
18.0000	0.0829	-0.0773	0.0416	-0.0426	-0.0000	0.0000	-0.0002	-0.0001
20.0000	0.0788	-0.0734	0.0395	-0.0403	-0.0000	0.0000	-0.0001	0.0000
22.0000	0.0750	-0.0703	0.0377	-0.0384	-0.0000	0.0000	-0.0001	0.0000
24.0000	0.0720	-0.0675	0.0361	-0.0368	0.0000	0.0000	-0.0000	0.0000
26.0000	0.0690	-0.0649	0.0347	-0.0353	0.0000	0.0000	-0.0000	0.0000
28.0000	0.0668	-0.0624	0.0334	-0.0340	0.0000	0.0000	-0.0000	0.0000
30.0000	0.0647	-0.0608	0.0323	-0.0328	0.0000	0.0000	0.0000	0.0000
32.0000	0.0623	-0.0592	0.0313	-0.0317	0.0000	0.0000	0.0000	0.0000
34.0000	0.0607	-0.0569	0.0304	-0.0307	-0.0000	-0.0000	-0.0000	-0.0000
36.0000	0.0586	-0.0533	0.0295	-0.0298	0.0000	0.0000	0.0000	0.0000
38.0000	0.0587	-0.0523	0.0287	-0.0291	-0.0000	-0.0000	-0.0000	-0.0000
40.0000	0.0559	-0.0511	0.0280	-0.0282	-0.0000	-0.0000	-0.0000	-0.0000
42.0000	0.0531	-0.0510	0.0273	-0.0277	-0.0000	-0.0000	-0.0000	-0.0000
44.0000	0.0521	-0.0496	0.0267	-0.0269	-0.0000	-0.0000	-0.0000	-0.0000
46.0000	0.0511	-0.0491	0.0261	-0.0264	-0.0000	-0.0000	-0.0000	-0.0000
48.0000	0.0480	-0.0462	0.0253	-0.0255	-0.0000	-0.0000	-0.0000	-0.0000
50.0000	0.0439	-0.0471	0.0248	-0.0251	-0.0000	-0.0000	-0.0000	-0.0000

Q = -1.000
 A = 1.000
 B = 1.500
 XO= 0.0
 XL= 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A
2.0000	0.2820	-0.7492	0.3863	-0.7587	-0.1600	-0.1428	-0.0565	0.3967
4.0000	0.2644	-0.3802	0.3771	-0.4306	-0.1415	-0.0130	-0.0130	0.2059
6.0000	0.1756	-0.2859	0.3619	-0.3336	-0.1330	-0.0747	-0.0270	0.1081
8.0000	0.0531	-0.3834	0.3445	-0.2878	-0.1428	0.0061	-0.0160	0.0566
10.0000	-0.4948	-1.0685	0.3507	-0.2597	-0.2131	-0.0566	-0.0143	0.0223
12.0000	-0.2636	0.6850	0.2812	-0.2742	0.0747	-0.0566	0.0022	0.0112
14.0000	-0.1195	0.2779	0.2651	-0.2477	0.0061	-0.0055	0.0010	0.0034
16.0000	-0.1295	0.1808	0.2510	-0.2331	-0.0022	-0.0010	0.0003	0.0025
18.0000	-0.1433	0.1471	0.2363	-0.2213	-0.0011	0.0003	0.0005	0.0012
20.0000	-0.1492	0.1367	0.2233	-0.2113	-0.0002	0.0002	0.0001	0.0007
22.0000	-0.1505	0.1326	0.2132	-0.2009	0.0001	0.0001	0.0000	0.0003
24.0000	-0.1464	0.1307	0.2051	-0.1936	0.0001	0.0000	0.0000	0.0001
26.0000	-0.1407	0.1273	0.1963	-0.1860	0.0001	0.0000	0.0000	0.0000
28.0000	-0.1346	0.1246	0.1895	-0.1796	0.0001	0.0000	0.0000	0.0000
30.0000	-0.1299	0.1208	0.1832	-0.1738	0.0000	0.0000	0.0000	0.0000
32.0000	-0.1253	0.1175	0.1761	-0.1692	0.0000	0.0000	0.0000	0.0000
34.0000	-0.1214	0.1142	0.1713	-0.1639	0.0000	0.0000	0.0000	0.0000
36.0000	-0.1183	0.1108	0.1666	-0.1600	0.0000	0.0000	0.0000	0.0000
38.0000	-0.1149	0.1080	0.1621	-0.1548	0.0000	0.0000	0.0000	0.0000
40.0000	-0.1118	0.1059	0.1577	-0.1520	0.0000	0.0000	0.0000	0.0000
42.0000	-0.1092	0.1031	0.1540	-0.1488	0.0000	0.0000	0.0000	0.0000
44.0000	-0.1065	0.1007	0.1514	-0.1448	0.0000	0.0000	0.0000	0.0000
46.0000	-0.1041	0.0989	0.1477	-0.1417	0.0000	0.0000	0.0000	0.0000
48.0000	-0.1021	0.0968	0.1436	-0.1392	0.0000	0.0000	0.0000	0.0000
50.0000	-0.0999	0.0947	0.1409	-0.1362	0.0000	0.0000	0.0000	0.0000

Q = -1.000
A = 2.000
B = 2.000
X0 = 0.0
XL = 1.000

W	Z11	Z22	Z12	V2/V1	TF.M	TF.A	TF.DB	TF.A				
2.0000	0.2754	-1.4598	0.3886	-1.4607	-0.1668	-1.4314	0.9261	-0.2890	0.9701	5.9807	-0.2636	342.6692
4.0000	0.2820	-0.7492	0.3863	-0.7587	-0.1600	-0.6994	0.7473	-0.4948	0.8962	5.6983	-0.9515	326.4893
6.0000	0.2819	-0.5072	0.3825	-0.5357	-0.1506	-0.4448	0.5440	-0.5992	0.8093	5.4495	-1.8383	312.2354
8.0000	0.2646	-0.3803	0.3771	-0.4306	-0.1415	-0.3076	0.3704	-0.6300	0.7308	5.2439	-2.7241	300.4521
10.0000	0.2267	-0.3103	0.3703	-0.3712	-0.1354	-0.2169	0.2478	-0.6174	0.6653	5.0941	-3.5399	291.8677
12.0000	0.1752	-0.2859	0.3621	-0.3336	-0.1330	-0.1510	0.1767	-0.5733	0.5999	5.0113	-4.4890	287.1287
14.0000	0.1184	-0.3075	0.3532	-0.3077	-0.1350	-0.1002	0.1366	-0.4915	0.5101	4.9834	-5.8464	285.5275
16.0000	0.0528	-0.3825	0.3440	-0.2880	-0.1426	-0.0564	0.0941	-0.3857	0.3970	4.9518	-8.0233	283.7158
18.0000	-0.0485	-0.5587	0.3397	-0.2718	-0.1623	-0.0079	0.0391	-0.2870	0.2897	4.8479	-10.7011	277.7637
20.0000	-0.4987	-1.0760	0.3511	-0.2610	-0.2143	0.1167	-0.0133	-0.2053	0.2058	4.6479	-13.7327	266.3044
22.0000	-1.8530	1.0967	0.3212	-0.3289	0.2478	0.2033	-0.0509	-0.1399	0.1489	4.3632	-16.5438	249.9906
24.0000	-0.2613	0.6844	0.2718	-0.2769	0.0743	-0.0272	-0.0709	-0.0816	0.1080	3.9971	-19.3287	229.0193
26.0000	-0.1389	0.3941	0.2710	-0.2590	0.0226	-0.0233	-0.0706	-0.0325	0.0777	3.5728	-22.1875	204.7064
28.0000	-0.1199	0.2781	0.2660	-0.2481	0.0061	-0.0159	-0.0563	0.0023	0.0563	3.1016	-24.9904	177.7083
30.0000	-0.1216	0.2167	0.2569	-0.2403	-0.0001	-0.0099	-0.0344	0.0197	0.0396	2.6209	-28.0368	150.1648
32.0000	-0.1294	0.1812	0.2498	-0.2339	-0.0022	-0.0056	-0.0145	0.0227	0.0269	2.1385	-31.4113	122.5281
34.0000	-0.1363	0.1600	0.2426	-0.2273	-0.0026	-0.0027	-0.0019	0.0178	0.0179	1.6792	-34.9250	96.2127
36.0000	-0.1431	0.1470	0.2363	-0.2211	-0.0022	-0.0010	0.0041	0.0112	0.0119	1.2225	-38.4603	70.0454
38.0000	-0.1477	0.1402	0.2300	-0.2161	-0.0016	-0.0001	0.0053	0.0057	0.0078	0.8207	-42.1103	47.0212
40.0000	-0.1505	0.1358	0.2236	-0.2114	-0.0010	0.0004	0.0050	0.0019	0.0054	0.3656	-45.3455	20.9484
42.0000	-0.1506	0.1343	0.2175	-0.2069	-0.0006	0.0004	0.0035	0.0002	0.0035	0.0672	-49.2005	3.8520
44.0000	-0.1502	0.1325	0.2133	-0.2021	-0.0003	0.0004	0.0025	-0.0006	0.0026	6.0409	-51.7389	346.1152
46.0000	-0.1483	0.1319	0.2085	-0.1976	-0.0001	0.0003	0.0013	-0.0009	0.0016	5.6734	-55.6857	325.0603
48.0000	-0.1460	0.1310	0.2039	-0.1934	0.0000	0.0002	0.0006	-0.0010	0.0011	5.2232	-58.7965	299.2690
50.0000	-0.1430	0.1294	0.2012	-0.1892	0.0001	0.0001	0.0001	-0.0008	0.0008	4.8878	-61.5897	280.0520

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